Linear Submodular Bandits
and their Application to Diversified Retrieval

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Abstract

Diversified retrieval and online learning are two core research areas in the design of modern information retrieval systems. In this paper, we propose the *linear submodular bandits problem*, which is an online learning setting for optimizing a general class of feature-rich submodular utility models for diversified retrieval. We present an algorithm, called LSBGREEDY, and prove that it efficiently converges to a near-optimal model. As a case study, we applied our approach to the setting of personalized news recommendation, where the system must recommend small sets of news articles selected from tens of thousands of available articles each day. In a live user study, we found that LSBGREEDY significantly outperforms existing online learning approaches.

1 Introduction

User feedback has become an invaluable source of training data for optimizing information retrieval systems in a rapidly expanding range of domains, most notably content recommendation (e.g., news, movies, ads). When designing retrieval systems that adapt to user feedback, two important challenges arise. First, the system should recommend optimally diversified content that maximizes coverage of the information the user finds interesting (to maximize positive feedback). Second, the system should make exploratory recommendations in order to learn a reliable model from feedback.

Challenge 1: diversification. In most retrieval settings, the retrieval system must recommend sets of articles, rather than individual articles. Furthermore, the recommended articles should be well diversified. This is motivated by the principle that recommending redundant articles leads to diminishing returns on utility, since users need to consume redundant information only once. This notion of diminishing returns is well-captured by submodular utility models, which have become an increasingly popular approach to modeling diversified retrieval tasks in recent years [24, 25, 18, 3, 21, 9, 16].

Challenge 2: feature-based exploration. In most retrieval settings, users typically only provide feedback on the articles recommended to them. This partial feedback issue leads to an inherent tension between exploration and exploitation when deciding which articles to recommend to the user. Furthermore, it is typically desirable to learn a feature-based model that can generalize to new or previously unseen articles and users; this is often called the contextual bandits problem [13, 15, 7].

Although there exist approaches that have addressed these challenges individually, to our knowledge there is no single approach which solves both simultaneously and is also practical to implement. For instance, existing online approaches for optimizing submodular functions typically assume a feature-free model, and thus cannot generalize easily [18, 22, 23]. Such approaches measure performance relative to the single best set (e.g., of articles). Thus, they are not suitable for many retrieval settings since the set of available articles can change frequently (e.g., news recommendation).

In this paper, we address both challenges in a unified framework. We propose the *linear submodular bandits problem*, which is an online learning setting for optimizing a general class of feature-based submodular utility models. To make learning practical, we represent the benefit of adding an article to an existing set of selected articles as a linear model with respect to the user's preferences. This class of models encompasses several existing information coverage utility models for diversified retrieval [24, 25, 9], and allows us to learn flexible models that can generalize to new predictions.

Similar to the contextual bandits setting considered in [15], our setting can be characterized as a feature-based exploration-exploitation problem, where the uncertainty lies in how best to model user interests using the available features. In contrast to [15], we aim to recommend optimally diversified sets of articles rather than just single articles. From that standpoint, modeling this additional layer of complexity in the bandit setting is our main technical contribution. We present an algorithm, called LSBGREEDY, to optimize this exploration-exploitation trade-off. When learning a d-dimensional model to recommend sets of L articles for T time steps, we prove that LSBGREEDY incurs regret that grows as $\mathcal{O}(d\sqrt{LT})$ (ignoring log factors). This regret matches the convergence rates of analogous algorithms for the conventional linear bandits setting [1, 20, 8].

As a case study, we applied our approach to the setting of personalized news recommendation [9, 15, 16]. In addition to simulation experiments, we conducted a live user study over a period of ten rounds, where in each round the retrieval system must recommend a small set of news articles selected from tens of thousands of available articles for that round. We compared against existing online learning approaches that either employ no exploration [9], or learn to recommend only single articles (and thus do not model diversity) [15]. Compared to previous approaches, we find that LSBGREEDY can significantly improve the performance of the retrieval system even when learning for a limited number of rounds. Our empirical results demonstrate the advantage of jointly tackling the challenges of diversification and feature-based exploration, as well as showcase the practicality of our approach.

2 Submodular Information Coverage Models

Before presenting our online learning setting, we first describe the class of utility functions that we optimize over. Throughout this paper, we use personalized news recommendation as our motivating example. In this setting, utility corresponds to the amount of interesting information covered by the set of recommended articles.

Suppose that news articles are represented using a set of d "topics" or "concepts" that we wish to cover (e.g., the Middle East or the weather).[1] Intuitively, recommending two articles that cover highly overlapping topics might not be more beneficial than recommending just one of the articles – this is the notion of diminishing returns we wish to capture in our information coverage model.

Two key properties we will exploit are that our utility functions are monotone and submodular. A set function F mapping sets of recommended articles A to real values (e.g., the total information covered by A) is monotone and submodular if and only if

$$F(A \cup \{a\}) \geq F(A) \quad \text{and} \quad F(A \cup \{a\}) - F(A) \geq F(B \cup \{a\}) - F(B),$$

respectively, for all articles a and sets $A \subseteq B$. In other words, since A is smaller than B, the benefit of adding a to A is larger than the benefit of adding a to B. Submodularity provides a natural framework for characterizing diminishing returns in information coverage, since the gain of adding a second (redundant) article on a topic will be smaller than the gain of adding the first.

For each topic i, let $F_i(A)$ be a monotone submodular function corresponding to how well the recommended articles A cover topic i. We write the total utility of recommending A as

$$F(A|w) = w^\top \langle F_1(A), \ldots, F_d(A) \rangle, \tag{1}$$

where $w \in \Re_+^d$ is a parameter vector indicating the user's interest level in each topic. Thus, $F(A|w)$ corresponds to the weighted information coverage of A, and depends on the preferences of the particular user. Since sums of monotone submodular functions are themselves monotone submodular, this implies that $F(A|w)$ is also monotone submodular (this would not hold if w has negative components). When making recommendations, the goal then is to select the A that maximizes $F(A|w)$. This class of information utility models encompasses several existing models of information coverage for diversified retrieval [24, 25, 9].

[1] In general, these features can represent any "nugget of information", such as a single word [24, 25, 9].

Example: Probabilistic Coverage. As an illustrative example, we now describe the probabilistic coverage model proposed in [9]. This will also be the coverage model used in our case study (see Section 5). Each article a has some probability $P(i|a)$ of covering topic i.[2] Assuming each article $a \in A$ has an independent probability of covering each topic, then we can write $F_i(A)$ as

$$F_i(A) = 1 - \prod_{a \in A} (1 - P(i|a)), \tag{2}$$

which corresponds to the probability that topic i is covered by at least one article in A. It is straightforward to check that F_i in (2) is monotone submodular [9].

Local Linearity. One attractive property of $F(A|w)$ in (1) is that the incremental gains are *locally linear*. In particular, the incremental gain of adding a to A can be written as $w^\top \Delta(a|A)$, where

$$\Delta(a|A) = \langle\, F_1(A \cup \{a\}) - F_1(A) \ , \ \ldots \ , \ F_d(A \cup \{a\}) - F_d(A) \,\rangle. \tag{3}$$

In other words, the i-th component of $\Delta(a|A)$ corresponds to the incremental coverage (i.e., submodular advantage) of topic i by article a, conditioned on articles A having already been selected. This property will be exploited by our online learning algorithm presented in Section 4.

Optimization. Another attractive property of monotone submodular functions is that the myopic greedy algorithm is guaranteed to produce a near-optimal solution [17]. For any budget L (e.g., $L = 10$ articles), the constrained optimization problem, $\text{argmax}_{A:|A| \leq L} F(A|w)$, can be solved greedily to produce a solution that is within a factor $(1 - 1/e) \approx 0.63$ of optimal. Achieving better than $(1 - 1/e)OPT$ is known to be intractable unless $P = NP$ [10]. In practice, the greedy algorithm can often perform much better than this worst case guarantee (cf. [14]), and will be a central component in our online learning algorithm.

3 Problem Formulation

We propose the *linear submodular bandits problem* which is described in the following. At each time step $t = 1, \ldots, T$, our algorithm interacts with the user in the following way:

- A set of articles $\mathcal{A}_t$ is made available to the algorithm. Each article $a \in \mathcal{A}_t$ is represented using a set of d basis coverage functions $F_1, \ldots, F_d$, defined as in Section 2, which is known to the algorithm.
- The algorithm chooses a ranked set of L articles, denoted $A_t = (a_t^{(1)}, \ldots, a_t^{(L)})$, using the basis coverage functions of the articles and the outcomes of previous time steps.
- The user provides feedback (e.g., clicks on or ignores each article), and the rewards for each recommended articles $r_t(A_t)$ (4) are observed.

In order to develop our algorithm, we require a model of user behavior. We assume the user scans the recommended articles $A = (a^{(1)}, \ldots, a^{(L)})$ one by one in top-down fashion. For each article $a^{(\ell)}$, the user considers the *new information* covered by $a^{(\ell)}$ and not covered by the above articles $A^{(1:\ell-1)}$ ($A^{(1:\ell)}$ denotes the articles in the first ℓ slots). In our representation, this new information is $\Delta(a^{(\ell)}|A^{(1:\ell-1)})$ as in (3). The user then clicks on (or likes) $a^{(\ell)}$ with independent probability $(w^*)^\top \Delta(a^{(\ell)}|A^{(1:\ell-1)})$, where w^* is the hidden preferences of the user. Formally, for any set of articles A chosen at time t, the rewards $r_t(A)$ can be written as the sum of rewards at each slot,

$$r_t(A) = \sum_{\ell=1}^{L} r_t^{(\ell)}(A). \tag{4}$$

We assume each $r_t^{(\ell)}$ is an independent random variable bounded in $[0, 1]$ and satisfies

$$\mathbf{E}\left[r_t^{(\ell)}(A)\right] = (w^*)^\top \Delta(a^{(\ell)}|A^{(1:\ell-1)}), \tag{5}$$

where w^* is a weight vector unknown to the algorithm with $\|w^*\| \leq S$. In other words, the expected reward in each slot is realizable, linear in $\Delta(a^{(\ell)}|A^{(1:\ell-1)})$, and independent of the other slots. We call this independence property *conditional submodular independence*, which we will leverage in

[2] E.g., the topics and coverage probabilities can be derived from a topic model such as LDA [4].

Algorithm 1 LSBGREEDY

```
1: input: $\lambda$, $\alpha_t$
2: for $t = 1, \ldots, T$ do
3:     $M_t \leftarrow \lambda I_d + \sum_{\tau=1}^{t-1} \sum_{\ell=1}^{L} \Delta_\tau^{(\ell)} \left(\Delta_\tau^{(\ell)}\right)^\top$   //covariance matrix
4:     $b_t \leftarrow \sum_{\tau=1}^{t-1} \sum_{\ell=1}^{L} \hat{r}_\tau^{(\ell)} \Delta_\tau^{(\ell)}$   //aggregate feedback so far
5:     $w_t \leftarrow M_t^{-1} b_t$   //linear regression using previous feedback as training data
6:     $A_t \leftarrow \emptyset$
7:     for $\ell = 1, \ldots, L$ do
8:         $\forall a \in \mathcal{A}_t \setminus A(t) : \mu_a \leftarrow w_t^\top \Delta\left(a|A_t\right)$   //compute mean estimate of utility gain
9:         $\forall a \in \mathcal{A}_t \setminus A(t) : c_a \leftarrow \alpha_t \sqrt{\Delta\left(a|A_t\right)^\top M_t^{-1} \Delta\left(a|A_t\right)}$   //compute confidence interval
10:        set $a_t^{(\ell)} \leftarrow \text{argmax}_a (\mu_a + c_a)$   //select article with highest upper confidence bound
11:        store $\Delta_t^{(\ell)} \leftarrow \Delta\left(a_t^{(\ell)} \middle| A_t^{(1:\ell-1)}\right)$, $A_t \leftarrow A_t \cup \left\{a_t^{(\ell)}\right\}$
12:    end for
13:    recommend articles $A_t$ in the order selected, and observe rewards $\hat{r}_t^{(1)}, \ldots, \hat{r}_t^{(L)}$ for each slot
14: end for
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our analysis. While conditional submodular independence may seem ideal, we will show in our user study experiments that it is not required for our proposed algorithm to achieve good performance.

Equations (4) and (5) imply that $\mathbf{E}[r_t(A)] = F(A|w^*)$ for F defined as in (1). Thus, $\mathbf{E}[r_t]$ is monotone submodular, and a clairvoyant system with perfect knowledge of w^* can greedily select articles to achieve (expected) reward at least $(1 - 1/e)OPT$, where OPT denotes the total expected reward of the optimal recommendations for $t = 1, \ldots, T$. Let A_t^* denote the optimal set of articles at time t. We quantify performance using the following notion of regret which we call *greedy regret*,

$$\text{Reg}_G(T) = \left(1 - \frac{1}{e}\right) \sum_{t=1}^{T} \mathbf{E}\left[r_t(A_t^*)\right] - \sum_{t=1}^{T} r_t(A_t) \equiv \left(1 - \frac{1}{e}\right) OPT - \sum_{t=1}^{T} r_t(A_t). \tag{6}$$

4 Algorithm and Main Results

A central question in the study of bandit problems is how best to balance the trade-off between exploration and exploitation (cf. [15]). To minimize regret (6), an algorithm must exploit its past experience to recommend sets of articles that appear to maximize information coverage. However, topics that appear good (i.e., interesting to the user) may actually be suboptimal due to imprecision in the algorithm's knowledge. In order to avoid this situation, the algorithm must explore by recommending articles about seemingly poor topics in order to gather more information about them.

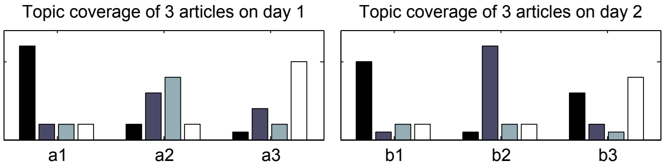
In this section, we present an algorithm, called LSBGREEDY, which automatically trades off between exploration and exploitation (Algorithm 1). LSBGREEDY balances exploration and exploitation using upper confidence bounds on the estimated gain in utility, and builds upon upper confidence bound style algorithms for the conventional linear bandits setting [8, 20, 15, 7, 1]. Intuitively, the algorithm can be decomposed into the following components.

Training a Model. Since we employ a linear model, at each time t, we can fit an estimate w_t of the true w^* via linear regression on the previous feedback. Lines 3–5 in Algorithm 1 describe this step, where $\Delta_\tau^{(\ell)}$ denotes the incremental coverage features of the article selected at time τ and slot ℓ, and $\hat{r}_\tau^{(\ell)}$ denotes the associated reward. Note that λ in Line 3 is the standard regularization parameter.

Estimating Incremental Coverage. Given w_t, we can now estimate the incremental gain of adding any article a to an existing set of results A. As discussed in Section 3, the true (expected) incremental gain is $(w^*)^\top \Delta(a|A)$. Our algorithm's estimate is $w_t^\top \Delta(a|A)$ (Line 8). If our algorithm were to purely exploit prior knowledge, then it would greedily choose articles that maximize $w_t^\top \Delta(a|A)$.[3]

Computing Confidence Intervals. Of course, each w_t is an imprecise estimate of the true w^*. Given such uncertainty, a natural approach is to use confidence intervals which contain the true w^*

[3] Note that w_t may have negative components, which would make $F(\cdot|w_t)$ not monotone submodular. However, regret is measured by $F(\cdot|w^*)$, which is monotone submodular. We show in our analysis that having negative components in w_t does not hinder our ability to converge efficiently to w^* in a regret sense.



t	$A_t^{(1)}$	$r_t^{(1)}$	$A_t^{(2)}$	$r_t^{(2)}$
1	a1	1	a2	0
2	b1	1	b3	1

Figure 1: Illustrative example of LSBGREEDY for $L = 2$ and 2 days. Each day comprises 3 articles covering 4 topics, which are depicted in the two plots. Each row in the table describes the choices of LSBGREEDY and the resulting feedback. In day 1, LSBGREEDY recommends articles to explore topics 1, 2, and 3, and the user indicates liking a1 and disliking a2. In day 2, LSBGREEDY recommends b1 to exploitatively cover topic 1, and b3 to both cover topic 1 and explore topic 4.

with some target confidence (e.g., 95%). Our algorithm's uncertainty in the gain of article a given set A depends directly to how much feedback we have collected regarding prominent topics in $\Delta(a|A)$. In our linear setting, uncertainty is measured using the inverse covariance matrix M_t^{-1} of the submodular features of the previously selected articles (Line 9). If our algorithm were to purely explore, then it would greedily select articles that have maximal uncertainty $\sqrt{\Delta(a|A)^\top M_t^{-1}\Delta(a|A)}$.

Balancing Exploration and Exploitation. In order to achieve low regret, LSBGREEDY greedily selects articles that maximize a compromise between estimated gain and uncertainty (Line 10), with α_t controlling the tradeoff. For any $\delta \in (0,1)$, Lemma 3 in Appendix A.2 provides sufficient conditions on α_t for constructing confidence intervals,

$$w_t^\top \Delta(a|A) \pm \alpha_t \sqrt{\Delta(a|A)^\top M_t^{-1}\Delta(a|A)} \equiv w_t^\top \Delta(a|A) \pm \alpha_t \|\Delta(a|A)\|_{M_t^{-1}}, \tag{7}$$

that contain the true value, $(w^*)^\top \Delta(a|A)$, with probability at least $1-\delta$. In this sense, Line 10 maximizes the *upper confidence bound* on the true expected reward. Figure 1 provides an illustrative example of the behavior of LSBGREEDY.

We now state our main result, which essentially bounds the greedy regret (6) of LSBGREEDY as $\mathcal{O}(d\sqrt{TL})$ (ignoring log factors). This means that the average loss incurred per slot and per day by LSBGREEDY relative to $(1-1/e)OPT$ decreases at a rate of $\mathcal{O}(d/\sqrt{TL})$.

Theorem 1. *For $L \leq d$, $\lambda = L$, and α_t defined as*

$$\alpha_t = \sqrt{2\log\left(2\det(M_t)^{1/2}\det(\lambda I_d)^{-1/2}/\delta\right)} + S\sqrt{\lambda}, \tag{8}$$

with probability at least $1-\delta$, LSBGREEDY *achieves greedy regret* (6) *bounded by*

$$\mathit{Reg}_G(T) \leq \alpha_T \sqrt{8TL\log\det(M_{T+1})} + \sqrt{2(1+TL)\log\left(\frac{\sqrt{1+TL}}{\delta/2}\right)} = \mathcal{O}\left(Sd\sqrt{TL}\log\left(\frac{TL}{\delta}\right)\right).$$

The proof of Theorem 1 is presented in Appendix A in the supplementary material. In practice, the choice of α_t in (8) may be overly conservative. As we show in our experiments, more aggressive choices of α_t can often lead to faster convergence.

5 Empirical Analysis: Case Study in News Recommendation

We applied LSBGREEDY to the setting of personalized news recommendation (cf. [9, 15, 16]), where the system is tasked with recommending sets of articles that maximally cover the interesting information of the available articles. The user provides feedback (e.g., by indicating that she likes or dislikes each article), and the goal is to maximize the total positive feedback by personalizing to the user. We conducted both simulation experiments as well as a live user study. Since real users are unlikely to behave exactly according to our modeling assumptions (e.g., obey conditional submodular independence), our user study tests the effectiveness of our approach in settings beyond those considered in our theoretical analysis.

5.1 Simulations

Data. We ran simulations using both synthetic datasets as well as the blog dataset from [9]. For each setting, we generated a hidden true preference vector w^*. For the synthetic data, all articles

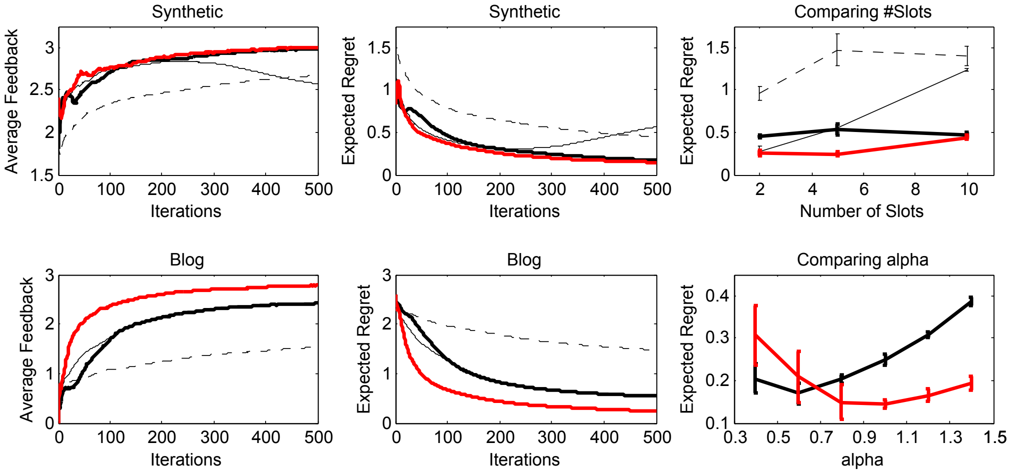


Figure 2: Simulation results comparing LSBGreedy (red), RankLinUCB (black thick), Multiplicative Weighting (black thin), and ϵ-Greedy (dashed thin). The middle column computes regret versus the clairvoyant greedy solution, and not $(1-1/e)OPT$. Unless specified, results are for $L=5$.

were randomly generated using $d=25$ topics, and w^* was randomly generated and re-scaled so the most likely articles were liked with probability $\approx 75\%$. For the blog dataset, articles are represented using $d=100$ topics generated using Latent Dirichlet Allocation [4], and w^* was derived from a preliminary version of our user study. Our simulated user behaves according to the user model described in Section 3. We use probabilistic coverage (2) as the submodular basis functions.

Competing Methods. We compared LSBGreedy against the following online learning algorithms. Note that all learning algorithms use the same underlying submodular utility model.

- Multiplicative Weighting (MW) as proposed in [9], which does not employ exploration.
- RankLinUCB, which combines the LinUCB algorithm [8, 20, 15, 7, 1] with Ranked Bandits [18, 22]. RankLinUCB is similar to LSBGreedy except that it maintains a separate weight vector per slot since it employs a reduction to L separate linear bandits (one per slot). In a sense, this is the natural application of existing approaches to our setting.[4]
- ϵ-Greedy, which randomly explores with probability ϵ, and exploits otherwise [15].

Results. Figure 2 shows a representative sample of our simulation results.[5] We see that both ϵ-Greedy and Multiplicative Weighting achieve significantly worse results than LSBGreedy. We also observe the performance of Multiplcative Weigthing diverge in the synthetic dataset, which is due to the fact that it does not employ exploration. RankLinUCB is more competitive, and achieves matching performance in the synthetic dataset. We also see that RankLinUCB is more sensitive to the choice of α. Interestingly, both LSBGreedy and RankLinUCB approach the same performance when recommending $L=10$ articles. This can be explained by the user's interests being saturated by 10 articles, and suggests that the bound in Theorem 1 could potentially be further refined. Additional details can be found in Appendix B in the supplementary material.

5.2 User Studies

Design. The design of our study is similar to the personalization study conducted in [9]. We presented each user with ten articles per day over ten days from January 18, 2009 to January 27, 2009. Each day, the articles are selected using an interleaving of two policies (described below). The articles are displayed as a title with its contents viewable via a preview pane. The user is instructed

[4] One can show that RankLinUCB achieves greedy regret (6) that grows as $\mathcal{O}(dL\sqrt{T})$ (ignoring log factors), which is a factor $\sqrt{L}$ worse than the regret guarantee of LSBGreedy.

[5] For all methods, we find performance to be relatively stable w.r.t. the tuning parameters (e.g., α_t for LSBGreedy). Unless specified, we set all parameters to values that achieve good results for their respective algorithms. In particular we set $\alpha_t=1$ for LSBGreedy, $\alpha_t=0.6$ for RankLinUCB, $\beta=0.9$ for MW, and $\epsilon=0.1$ for ϵ-Greedy. LSBGreedy, RankLinUCB, and ϵ-Greedy train linear models with regularization parameter λ, which we kept constant at $\lambda=1$.

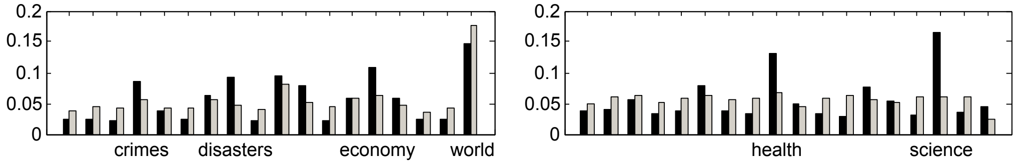


Figure 3: Displaying normalized learned preferences of LSBGREEDY (dark) and MW (light) for two user study sessions. In the left session, MW overfits to the "world" topic. In the right session, the user likes very few articles, and MW does not discover any topics that interest the user.

COMPARISON	#SESSIONS	WIN/TIE/LOSE	GAIN PER DAY	% OF LIKES
LSBGREEDY vs Static Baseline	24	24 / 0 / 0	1.07	63% (67%)
LSBGREEDY vs Mult. Weighting	26	24 / 1 / 1	0.54	57% (63%)
LSBGREEDY vs RankLinUCB	27	21 / 2 / 4	0.58	57% (61%)

Table 1: User study comparing LSBGREEDY with competing algorithms. The parenthetical values in the last column are computed ignoring clicks on articles jointly recommended by both algorithms (see Section 5.2). All results are statistically significant with 95% confidence.

to briefly skim each article to get a sense of its content and, one by one, mark each article as "interested in reading in detail" (like), or "not interested" (dislike). As in [9], for each decision, the user is told to take into account the articles shown above in the current day, so as to capture the notion of incremental coverage. For example, a user might be interested in reading an article regarding the Middle East appearing at the top slot, and would mark it as "interested." However, if several very similar articles appear below it, the user may mark the subsequent articles as "not interested."

Evaluation. For each day, we generate an interleaving of recommendations from two algorithms. Interleaving allows us to make paired comparisons such that we simultaneously control for the particular user and particular day (certain days may contain more or less interesting content to the user than other days). Like other interleaving approaches [19], our approach maintains a notion of fairness so that both competing algorithms recommend the same amount of content. After each day, the user's feedback is collected and given to the two competing algorithms. Additional details of our experimental setup can be found in Appendix C in the supplementary material.

Data. In order to distinguish the gains of the algorithms from other effects (such as imperfections in the features, or having too high a dimension to converge), we performed dimensionality reduction. We created 18 genres (examples shown in Figure 3), labeled relevant articles and trained a model via linear regression for each genre. Note that many articles are relevant to multiple genres.

We compared LSBGREEDY against the static baseline (i.e., no personalization), Multiplicative Weighting (MW) from [9], and RankLinUCB. We evaluated each comparison setting using approximately twenty five participants, most of whom are graduate students or young professionals.

Results. Table 1 describes our results. We first aggregated per user, and then aggregated over all users. For each user, we computed three statistics: (1) whether LSBGREEDY won, tied, or lost in terms of total number of liked articles, (2) the difference in liked articles per day, and (3) the fraction of liked articles recommended by LSBGREEDY. Jointly recommended articles can be either counted as half to each algorithm or ignored (these results are shown in parentheticals in Table 1).

Overall, about 90% of users preferred recommendations by LSBGREEDY over the competing algorithms. On average, LSBGREEDY obtains about one additional liked article per day and 63% of all liked articles versus the static baseline, and about half an additional liked article per day and 57% of all liked articles versus the two competing learning algorithms. The gains we observe are all statistically significant with 95% confidence, and show that LSBGREEDY can be effective even when the assumptions in our theoretical analysis may not be satisfied.

Figure 3 shows the learned preferences by LSBGREEDY and MW on two sessions. Since MW does not employ exploration, it can either overfit to its previous experience and not find new topics that interest the user (left plot), or fail to discover any good topics (right plot). We do not include a comparison with RankLinUCB since it learns L preference vectors, which are difficult to visualize.

6 Related Work

Diversified Retrieval. We are chiefly interested in training flexible submodular utility models, since such models yield practical algorithmic approaches. At one extreme are feature-free models that do not require training. However, such models are limited to unpersonalized settings that ignore context, such as recommending a global set of blogs to monitor [14]. On the other hand, methods that use feature-rich models typically either employ unsupervised training [24] or require fine-grained subtopic labels [25]. Such learning approaches cannot easily adapt to new domains. One exception is [9], whose proposed online learning approach does not incorporate exploration. As shown in our experiments, this significantly inhibits the learning ability of their approach.

Beyond submodular models of information coverage, other approaches include methods that balance relevance and novelty [5, 26, 6] and graph-based methods [27]. For such models, it remains a challenge to design provably efficient online learning algorithms.

Bandit Learning. From the perspective of our work, existing bandit approaches can be categorized along two dimensions: single-prediction versus set-prediction, and feature-based versus feature-free.

Most feature-based settings are designed to predict single results, rather than sets of results. Of such settings, the most relevant to ours is the linear stochastic bandits setting [8, 20, 15, 7, 1], which we build upon in our approach. One limitation here is the assumption of realizability – that the "true" user model lies within our class. It may be possible to develop more robust algorithms for our submodular bandits setting by building upon algorithms with more general guarantees (e.g., [2]).

Most set-based settings, such as bandit submodular optimization or the general bandit slate problem, assume a feature-free model [18, 22, 23, 12]. As such, performance is quantified relative to a fixed set of articles, which is not appropriate for many retrieval settings (e.g., news recommendation). One exception is [21], which assumes that document and user models lie within a metric space. However, it is unclear how to incorporate our submodular features into their setting.

7 Discussion of Limitations and Future Work

Submodular Basis Features. Our approach requires access to submodular basis functions as features. In practice these basis features are often derived using various topic modeling or dimensionality reduction techniques. However, the resulting features are almost always noisy or biased. Furthermore, one expects that different users will be better modeled using different basis features. As such, one important direction for future work is to learn the appropriate basis features from user feedback, which is similar to the setting of interactive topic modeling [11].

Moreover, user behavior is likely to be influenced by many factors beyond those well-modeled by submodular basis features. For example, the probability of the user liking a certain article could be influenced by the time of day, or day of the week. A more unified approach would be to incorporate both these standard features as well as submodular basis features in a joint model.

Curse of Dimensionality. The convergence rate of LSBGREEDY depends linearly on the number of features d (which appears unavoidable without further assumptions). Thus, our approach may not be practical for settings that use a very large number of features. One possible extension is to jointly learn from multiple users simultaneously. If users tend to have similar preferences, then learning jointly from multiple users may yield convergence rates that are sub-linear in d.

8 Conclusion

We proposed an online learning setting for optimizing a general class of submodular functions. This setting is well-suited for modeling diversified retrieval systems that interactively learn from user feedback. We presented an algorithm, LSBGREEDY, and proved that it efficiently converges to a near-optimal model. We conducted simulations as well as user studies in the setting of news recommendation, and found that LSBGREEDY outperforms competing online learning approaches.

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Supplementary Material

A Theoretical Analysis

Our analysis follows three conceptual phases:

- In the first phase, we present a reduction to a "parallel" bandit setting by introducing slot-wise regret defined in (10) below. We will show that the sum of slot-wise regret upper bounds the greedy regret (6), thus making it sufficient to bound the sum of slot-wise regret.
- In the second phase, we show that our confidence intervals parameterized by α_t defined in (8) contain the the true expected value of every possible $\Delta(a|A) \in \Re_+^d$ (for any articles a and sets A) with sufficient probability. This will be used to show that the sum of slot-wise regret depends directly on the size of the confidence intervals.
- In the third phase, we show that our confidence intervals shrink sufficiently fast to yield regret sublinear in both T and L (when $L \leq d$).

We first introduce the notion of the slot-wise optimal reward $OPT^{(\ell)}$. For each slot ℓ and time t, we assume that the choices of slots 1 to $\ell - 1$ are fixed to be the choices of LSBGreedy, $A_t^{(1:\ell-1)}$. The ℓ-th slot optimal reward relative to $A_1^{(1:\ell-1)}, \ldots, A_T^{(1:\ell-1)}$ is then defined as

$$OPT^{(\ell)} = \sum_{t=1}^{T} \max_{a \in \mathcal{A}_t \setminus A_t^{(1:\ell-1)}} (w^*)^\top \Delta(a|A_t^{(1:\ell-1)}) \equiv \sum_{t=1}^{T} r_t^{(\ell)}(\tilde{a}_t^{(\ell)}|A_t^{(1:\ell-1)}), \tag{9}$$

where $\tilde{a}_t^{(\ell)}$ above is defined to be the locally optimal article at time t and slot ℓ conditioned on $A_t^{(1:\ell-1)}$. One can interpret this to be the best possible choice at slot ℓ conditioned on fixing the choices of slots $1, \ldots, \ell - 1$. We can thus also define the slot-wise regret correspondingly. In particular, the ℓ-th slot regret, conditioned on $A_1^{(1:\ell-1)}, \ldots, A_T^{(1:\ell-1)}$, is

$$\begin{aligned} \mathrm{Reg}^{(\ell)}(T) &= \sum_{t=1}^{T} r_t^{(\ell)}(\tilde{a}_t^{(\ell)}|A_t^{(1:\ell-1)}) - \sum_{t=1}^{T} r_t^{(\ell)}(a_t^{(\ell)}|A_t^{(1:\ell-1)}) \\ &\equiv OPT^{(\ell)} - \sum_{t=1}^{T} r_t^{(\ell)}(a_t^{(\ell)}|A_t^{(1:\ell-1)}), \end{aligned} \tag{10}$$

where $a_t^{(\ell)}$ denotes the article chosen by LSBGreedy for slot ℓ and time t. In essense, slot-wise regret quantifies the local (per-slot) regret incurred by LSBGreedy, conditioned on its choices in previous slots. Note that the above definitions of slot-wise optimal reward (9) and slot-wise regret (10) can be defined for any online algorithm for the linear submodular bandits setting.

A.1 Reduction to Slot-wise Regret

We now show that the slot-wise regret (10) upper bounds the greedy regret (6). We then bound the slot-wise regret in Appendix A.2. The following analysis builds upon the setting considered in [18, 22].

Lemma 1. *For all L slot-wise regret $Reg^{(\ell)}(T)$ defined in* (10)*, we have*

$$\begin{aligned} \sum_{t=1}^{T} \hat{r}_t^{(\ell)} &\equiv \sum_{t=1}^{T} \left[r_t(A_t^{(1:\ell)}) - r_t(A_t^{(1:\ell-1)}) \right] \\ &\geq \frac{1}{L} \sum_{t=1}^{T} \left[\mathbf{E}[r_t(A_t^*)] - r_t(A_t^{(1:\ell-1)}) \right] - Reg^{(\ell)}(T) \\ &= \frac{1}{L} \left(OPT - \sum_{t=1}^{T} r_t(A_t^{(1:\ell-1)}) \right) - Reg^{(\ell)}(T) \end{aligned} \tag{11}$$

Proof. We know that the reward functions $r_t(A)$ are monotone submodular set functions that can be written as

$$\mathbf{E}[r_t(A)] = \sum_{\ell=1}^{|A|} (w^*)^\top \Delta(a^{(\sigma(\ell))} | A^{(\sigma(1:\ell))}),$$

for any permutation σ on $\{1, \ldots, |A|\}$, where $\sigma(1:\ell) = \{\sigma(1), \ldots, \sigma(\ell)\}$.

For any time t, by the pigeonhole principle, there must exist an article $a \in A_t^* \setminus A_t^{(1:\ell-1)}$ such that

$$\begin{aligned} \mathbf{E}[r_t^{(\ell)}(a|A_t^{(1:\ell-1)})] &\equiv \mathbf{E}[r_t(A_t^{(1:\ell-1)} \cup \{a\})] - \mathbf{E}[r_t(A_t^{(1:\ell-1)})] \\ &\geq \frac{1}{L}\left(\mathbf{E}[r_t(A_t^*)] - \mathbf{E}[r_t(A_t^{(1:\ell-1)})]\right). \end{aligned}$$

This implies that

$$\begin{aligned} \mathbf{E}[r_t^{(\ell)}(\tilde{a}_t^{(\ell)}|A_t^{(1:\ell-1)})] &\equiv \mathbf{E}[r_t(A_t^{(1:\ell-1)} \cup \{\tilde{a}_t^{(\ell)}\})] - \mathbf{E}[r_t(A_t^{(1:\ell-1)})] \\ &\geq \frac{1}{L}\left(\mathbf{E}[r_t(A_t^*)] - \mathbf{E}[r_t(A_t^{(1:\ell-1)})]\right), \end{aligned}$$

since $\tilde{a}_t^{(\ell)}$ is the article that maximizes $r_t^{(\ell)}(\cdot|A_t^{(1:\ell-1)})$. Noting that

$$\sum_{t=1}^{T} \hat{r}_t^{(\ell)} = \sum_{t=1}^{T} \mathbf{E}[r_t^{(\ell)}(\tilde{a}_t^{(\ell)}|A_t^{(1:\ell-1)})] - \text{Reg}^{(\ell)}(T)$$

completes the proof. □

Lemma 2. *For any algorithm in the linear submodular bandits setting predicting A_t at time t, we have*

$$\sum_{t=1}^{T} r_t(A_t) \geq \left(1 - \frac{1}{e}\right) OPT - \sum_{\ell=1}^{L} Reg^{(\ell)}(T),$$

thus implying that greedy regret (6) *is bounded by the sum of slot-wise regrets* (10)*,*

$$Reg_G(T) \leq \sum_{\ell=1}^{L} Reg^{(\ell)}(T).$$

Proof. We prove by induction on the slots ℓ that

$$OPT - \sum_{t=1}^{T} r_t(A_t^{(1:\ell)}) \leq \left(1 - \frac{1}{L}\right)^{\ell} OPT + \sum_{m=1}^{\ell} \text{Reg}^{(\ell)}(T). \tag{12}$$

The result of the lemma follows by taking $\ell = L$ and using the inequality $(1 - 1/L)^L < 1/e$.

The base case $\ell = 0$ is trivial. In the inductive case, let

$$\begin{aligned} Z^{(\ell)} &= OPT - \sum_{t=1}^{T} r_t(A_t^{(1:\ell)}) \\ &\equiv OPT - \sum_{m=1}^{\ell} \sum_{t=1}^{T} \hat{r}_t^{(m)}. \end{aligned}$$

Thus, we have

$$Z^{(\ell)} = Z^{(\ell-1)} - \sum_{t=1}^{T} \hat{r}_t^{(\ell)}. \tag{13}$$

Lemma 1 implies that

$$\sum_{t=1}^{T} \hat{r}_t^{(\ell)} \geq \frac{1}{L} Z^{(\ell-1)} - \text{Reg}^{(\ell)}(T). \tag{14}$$

Combining (13) and (14) yields

$$Z^{(\ell)} \leq \left(1 - \frac{1}{L}\right) Z^{(\ell-1)} + \text{Reg}^{(\ell)}(T).$$

Applying the inductive hypothesis proves (12). □

A.2 Bounding Slot-wise Regret

In this section, we build upon previous results for linear stochastic bandit optimization [1, 20, 8] to the parallel setting in order to bound the sum of slot-wise regret (10) of LSBGREEDY. Due to Lemma 2, this will also bound the greedy regret (6).

The following lemma provides a confidence region for the expected value every x (i.e. $(w^*)^\top x$) given our current estimate w_t.

Lemma 3 (Minor modification of Theorem 8 in [1]). *For rewards bounded in* $[0, 1]$, *we have*

$$\forall t, \ell, x : \|w_t^\top x - (w^*)^\top x\| \le \|x\|_{M_t^{-1}} \left(\sqrt{2 \log \left(\frac{\det(M_t)^{1/2} \det(\lambda I_D)^{-1/2}}{\delta} \right)} + S\sqrt{\lambda} \right),$$

with probability at least $1 - \delta$.

Lemma 3 follows from Theorem 8 in [1] by observing that, at each time t and slot ℓ, and for any a, the deviation of the reward $r_t(a|A_t^{(1:\ell-1)})$ from its expectation $(w^*)^\top \Delta(a|A_t^{(1:\ell-1)})$ behaves like a sub-Gaussian martingale difference (with scale factor 1). This argument follows since the rewards at each slot are sampled independently due to our assumption of conditional submodular independence. For the actions chosen by LSBGREEDY, let each $\epsilon_t^{(\ell)}$ be the random variable corresponding to the deviation of the realized reward $\hat{r}_t^{(\ell)}$ from its expectation, i.e.

$$\epsilon_t^{(\ell)} = (w^*)^\top \Delta_t^{(\ell)} - \hat{r}_t^{(\ell)}, \qquad (15)$$

where $\Delta_t^{(\ell)} = \Delta(a_t^{(\ell)}|A_t^{(1:\ell-1)})$ denotes the submodular features of the article chosen for time t and slot ℓ (as also described Line 11 in Algorithm 1). Again, we note that the sequence $\epsilon_t^{(\ell)}$ (ordered following $\epsilon_t^{(1)} \ldots, \epsilon_t^{(L)}$) is a conditionally sub-Gaussian martingale difference (with scale factor 1). The following lemma provides a bound on the sum of deviation errors.

Lemma 4. *For* $\epsilon_t^{(\ell)}$ *defined in* (15), *we have*

$$P\left(\sum_{t=1}^{T} \sum_{\ell=1}^{L} \epsilon_t^{(\ell)} > \sqrt{2(1 + TL) \log \left(\frac{\sqrt{1 + TL}}{\delta} \right)} \right) \le \delta.$$

Lemma 4 is a straightforward modification of Theorem 3 and Corollary 1 in [1]. It is essentially a one-dimensional version of Lemma 3.

Lemma 5. *If* $\lambda \ge L$, *then we have*

$$\sum_{t=1}^{T} \sum_{\ell=1}^{L} \|\Delta_t^{(\ell)}\|_{M_t^{-1}}^2 \le 2 \log \det(M_{T+1}).$$

Proof. Adapting from Lemma 4 in [1], we have

$$\begin{aligned}
\sum_{t=1}^{T} \sum_{\ell=1}^{L} \|\Delta_t^{(\ell)}\|_{M_t^{-1}}^2 &\le 2 \sum_{t=1}^{T} \log \left(1 + \sum_{\ell=1}^{L} \|\Delta_t^{(\ell)}\|_{M_t^{-1}}^2 \right) && (16) \\
&\le 2 \sum_{t=1}^{T} \log \det \left(I_d + \left(M_t^{-1/2} \sum_{\ell=1}^{L} \Delta_t^{(\ell)} \right) \left(M_t^{-1/2} \sum_{\ell=1}^{L} \Delta_t^{(\ell)} \right)^\top \right) && (17) \\
&= 2 \sum_{t=1}^{T} \left[\log \det \left(M_t + \left(\sum_{\ell=1}^{L} \Delta_t^{(\ell)} \right) \left(\sum_{\ell=1}^{L} \Delta_t^{(\ell)} \right)^\top \right) - \log \det (M_t) \right] \\
&= 2 \sum_{t=1}^{T} \left[\log \det (M_{t+1}) - \log \det (M_t) \right] \\
&\le 2 \log \det (M_{T+1}),
\end{aligned}$$

where (16) holds since $2\log(1+x) \geq x$ for $x \in [0,1]$ and $\|\Delta_t^{(\ell)}\|^2_{M_t^{-1}} \leq 1/\lambda \leq 1/L$, and (17) holds since, for any $X \in \Re_+^{d \times L}$, the determinant

$$\det\left(I_d + XX^T\right)$$

is minimized when XX^T is rank 1, i.e.,

$$\det\left(I_d + XX^T\right) \geq 1 + \sum_{\ell=1}^{L} \|X^{(\ell)}\|^2,$$

where $X^{(\ell)}$ denotes the ℓ-th column of X. □

Proof of Theorem 1. We can write the sum of slot-wise regret (10) as

$$\begin{aligned}
\sum_{\ell=1}^{L} \mathrm{Reg}^{(\ell)}(T) &= \sum_{t=1}^{T}\sum_{\ell=1}^{L} (w^*)^\top \Delta(\tilde{a}_t^{(\ell)} | A_t^{(1:\ell-1)}) - (w^*)^\top \Delta_t^{(\ell)} + \epsilon_t^{(\ell)} \\
&\leq \sum_{t=1}^{T}\sum_{\ell=1}^{L} w_t^\top \Delta_t^{(\ell)} + \alpha_t \|\Delta_t^{(\ell)}\|_{M_t^{-1}} - (w^*)^\top \Delta_t^{(\ell)} + \epsilon_t^{(\ell)} \qquad (18) \\
&\leq \sum_{t=1}^{T}\sum_{\ell=1}^{L} 2\alpha_t \|\Delta_t^{(\ell)}\|_{M_t^{-1}} + \epsilon_t^{(\ell)}
\end{aligned}$$

where (18) holds with probability at least $1 - \delta/2$. This follows from Lemma 3 and the fact that LSBGREEDY always picks the article whose features $\Delta_t^{(\ell)}$ maximizes

$$w_t^\top \Delta_t^{(\ell)} + \alpha_t \|\Delta_t^{(\ell)}\|_{M_t^{-1}}$$

as indicated in Lines 10–11 in Algorithm 1. Lemma 4 implies that, with probability $1 - \delta/2$,

$$\sum_{t=1}^{T}\sum_{\ell=1}^{L} \epsilon_t^{(\ell)} \leq \sqrt{2(1+TL)\log\left(\frac{\sqrt{1+TL}}{\delta/2}\right)}.$$

Thus, by the union bound, we know that

$$\sum_{\ell=1}^{L} \mathrm{Reg}^{(\ell)}(T) \leq \sum_{t=1}^{T}\sum_{\ell=1}^{L} 2\alpha_t \|\Delta_t^{(\ell)}\|_{M_t^{-1}} + \sqrt{2(1+TL)\log\left(\frac{\sqrt{1+TL}}{\delta/2}\right)}$$

holds with probability at least $1 - \delta$.

We know that the regret incurred at each time t and slot ℓ is at most 1. We can thus write the sum of slot-wise regret as

$$\begin{aligned}
\sum_{\ell=1}^{L} \mathrm{Reg}^{(\ell)}(T) &\leq \sqrt{TL \sum_{t=1}^{T}\sum_{\ell=1}^{L} \min\left(2\alpha_t \|\Delta_t^{(\ell)}\|_{M_t^{-1}}, 1\right)^2} + \sqrt{2(1+TL)\log\left(\frac{\sqrt{1+TL}}{\delta/2}\right)} \\
&\leq \alpha(T)\sqrt{4TL \sum_{t=1}^{T}\sum_{\ell=1}^{L} \|\Delta_t^{(\ell)}\|^2_{M_t^{-1}}} + \sqrt{2(1+TL)\log\left(\frac{\sqrt{1+TL}}{\delta/2}\right)}.
\end{aligned}$$

Applying Lemma 5 and Lemma 2 yields

$$\mathrm{Reg}_G(T) \leq \alpha_T \sqrt{8TL \log\det(M_{T+1})} + \sqrt{2(1+TL)\log\left(\frac{\sqrt{1+TL}}{\delta/2}\right)}.$$

Noting that

$$\log\det\left(M_{T+1}\right) \leq d\log(L(1+T/d))$$

and that $L \leq d$ yields

$$\begin{aligned}
\mathrm{Reg}_G(T) &\leq \left(\sqrt{2d\log\left(\frac{1+T/d}{\delta/2}\right)} + S\sqrt{d}\right)\sqrt{8dTL\log(L(1+T/d))} + \sqrt{2(1+TL)\log\left(\frac{\sqrt{1+TL}}{\delta/2}\right)} \\
&= \mathcal{O}\left(Sd\sqrt{TL}\log\left(\frac{TL}{\delta}\right)\right).
\end{aligned}$$

□

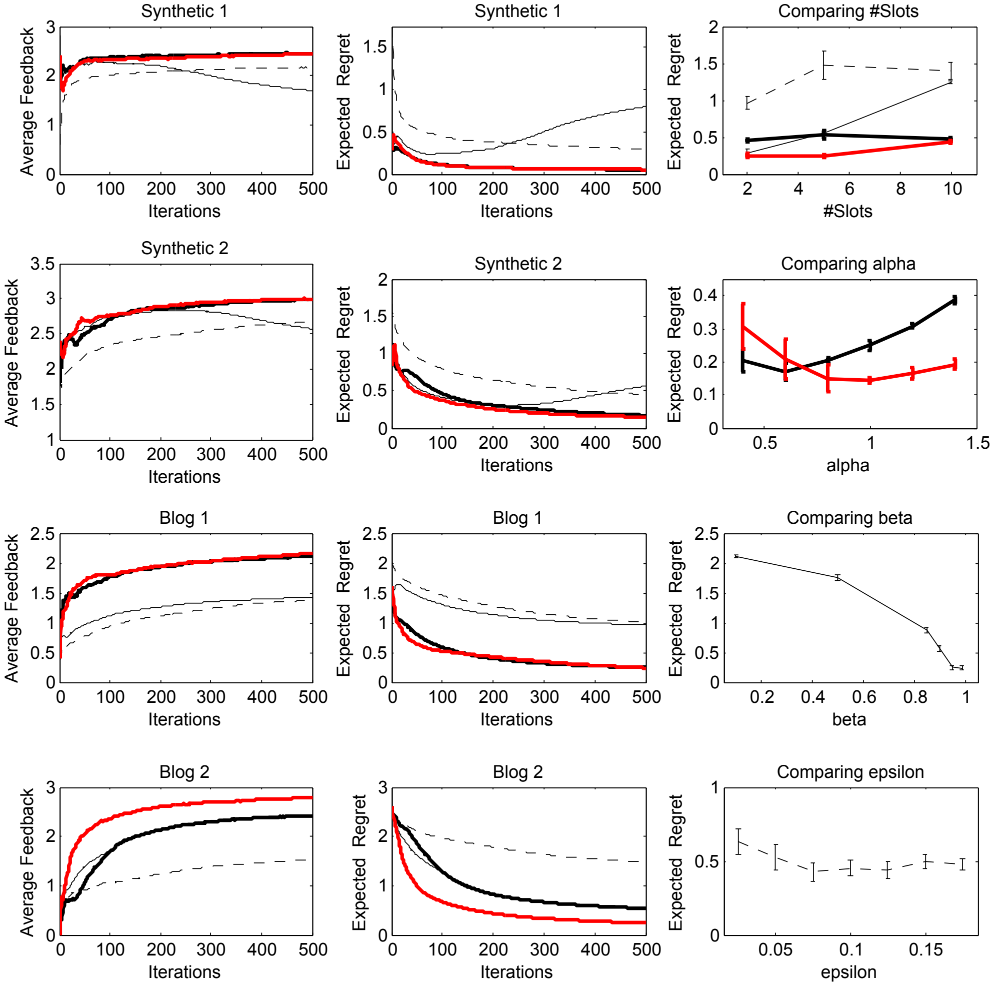


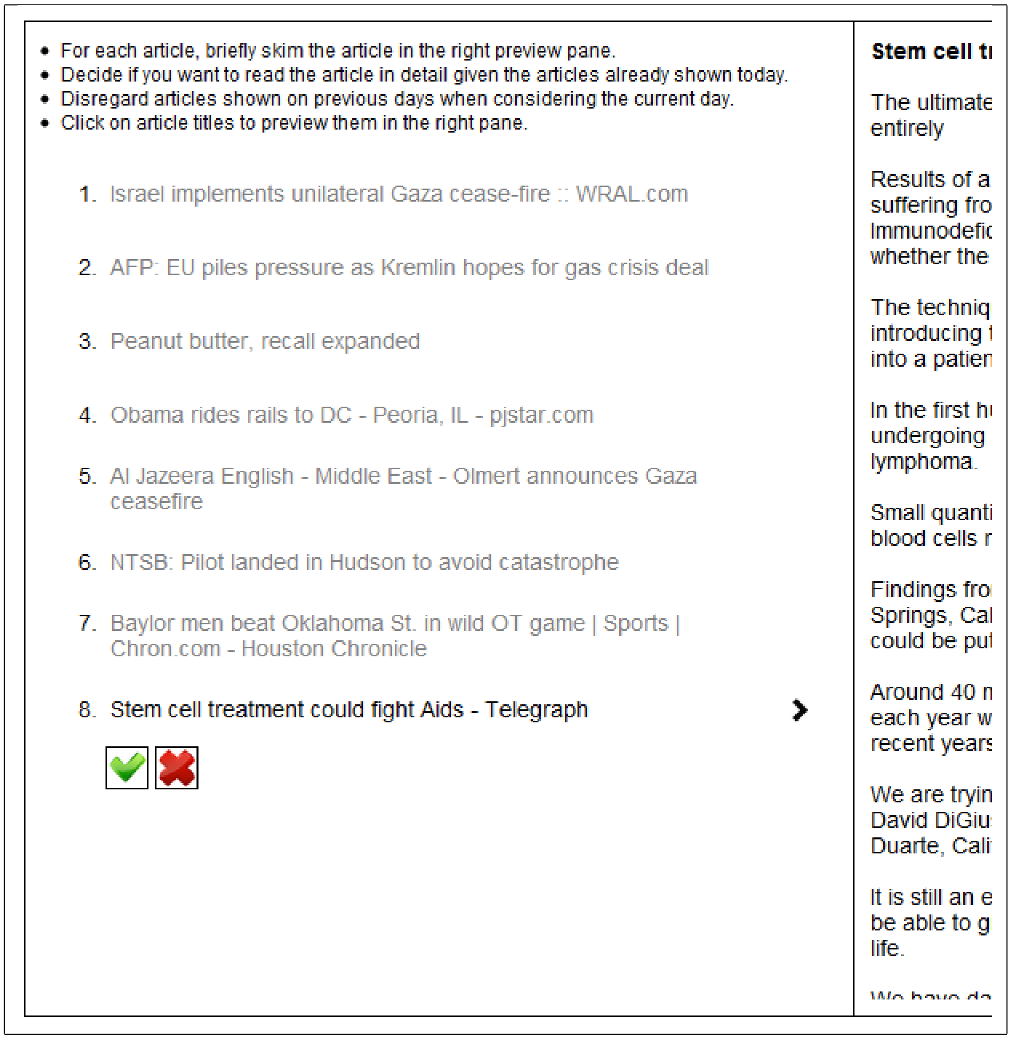
Figure 4: Simulation results comparing LSBGreedy (red), RankLinUCB (black thick), Multiplicative Weighting (black thin), and ϵ-Greedy (dashed thin). Synthetic 2 and Blog 2 are identical to Synthetic and Blog, respectively, from Figure 2. The middle column computes regret versus the clairvoyant greedy solution, and not $(1 - 1/e)OPT$. Unless specified, results are for $L = 5$.

B Simulation: Additional Details

Lazy Algorithm. For our experiments, we use a lazy version of LSBGreedy which does not evaluate every possible article for selection at each slot. In essence, we replace Lines 7-12 in Algorithm 1 with the CELF algorithm [14]. Note that Lines 7-12 optimize for a non-submodular objective due to the variance term (Line 9), meaning the theoretical guarantees of CELF no longer apply. Nonetheless, this leads to a dramatic speedup in computational performance with no noticeable difference in solution quality. We also use CELF to optimize for RankLinUCB and Multiplicative Weighting.

Simulation Results. Figure 4 shows our results. The right column shows results as parameters are varied. The top two results are identical the right column in Figure 2. We see that Multiplicative Weighting benefits from higher β, and that ϵ-Greedy does not seem very affected by the choice of ϵ.

Multiplicative Weighting (MW) does not employ exploration and so runs the risk of diverging away from w^* instead of converging. When MW diverges, it typically converges to a corner in the space of preference vectors (i.e., one component in w_t is 1 and the rest are 0). This is due to the fact that the most liked topic by the user will receive exponentially greater weighting as a result of the MW update rule (see [9] for details). One reason that MW performs best using larger β is that larger values of β correspond to smaller updates. This causes MW to diverge more slowly.



- For each article, briefly skim the article in the right preview pane.
- Decide if you want to read the article in detail given the articles already shown today.
- Disregard articles shown on previous days when considering the current day.
- Click on article titles to preview them in the right pane.

1. Israel implements unilateral Gaza cease-fire :: WRAL.com
2. AFP: EU piles pressure as Kremlin hopes for gas crisis deal
3. Peanut butter, recall expanded
4. Obama rides rails to DC - Peoria, IL - pjstar.com
5. Al Jazeera English - Middle East - Olmert announces Gaza ceasefire
6. NTSB: Pilot landed in Hudson to avoid catastrophe
7. Baylor men beat Oklahoma St. in wild OT game | Sports | Chron.com - Houston Chronicle
8. Stem cell treatment could fight Aids - Telegraph ›

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Figure 5: User study interace.

In several settings, we observe that RankLinUCB matches or nearly matches the performance of LSBGreedy. It remains an open question to determine how best to characterize the difficulty of problem instances to determine when LSBGreedy enjoys a clear advantage over RankLinUCB.

C User Study: Additional Details

Interleaved Evaluation. For each day, we generate an interleaving of recommendations from two policies. Algorithm 2 describes our interleaving algorithm, which is similar to previously proposed interleaving algorithms for evaluating retrieval functions via user feedback [19]. In contrast to previous approaches, Algorithm 2 allows the two retrieval policies to change their recommendations given the recommendations of the other policy, which is crucial for policies that make selections based on incremental coverage. As a paired evaluation, interleaving offers the following advantages:

- Interleaving is a blind test that preserves fairness in the sense that both polices own approximately the same number of results amongst the top ℓ for all $\ell = 1, \ldots, L$.
- The available articles on different days may contain differing amounts of interesting information to different users. Interleaving allows us to couple the two algorithms together on each day to control for a various extraneous sources of variation.
- One alternative is to show results from the same day for two consecutive iterations, with the two policies alternating per iteration. However, with a budget of ten iterations, this would reduce the learning period of each policy to five days. Interleaving allows each policy to learn over all ten days.

Algorithm 2 Interleaving algorithm for combining recommendations from two greedy policies.

```
input: recommender polices $F$, $G$
input: set of articles $\mathcal{A}$
$A \leftarrow \emptyset,\ n_F \leftarrow 0,\ \ n_G \leftarrow 0$
for $\ell = 1, \dots, L$ do
    $a_F \leftarrow \text{argmax}_{a \in \mathcal{A} \setminus A} F(A \cup \{a\})$  //policy F computes its greedy recommedation
    $a_G \leftarrow \text{argmax}_{a \in \mathcal{A} \setminus A} G(A \cup \{a\})$  //policy G computes its greedy recommedation
    if $a_F = a_G$ then
        $A \leftarrow A \cup \{a_F\},\ n_F \leftarrow n_F + 1,\ n_G \leftarrow n_G + 1$  //the two policies share ownership of this result
    else if $n_F < n_G$ then
        $A \leftarrow A \cup \{a_F\},\ n_F \leftarrow n_F + 1$  //policy F owns this result
    else if $n_G < n_F$ then
        $A \leftarrow A \cup \{a_G\},\ n_G \leftarrow n_G + 1$  //policy G owns this result
    else
        if $randombit = 1$ then
            $A \leftarrow A \cup \{a_F\},\ n_F \leftarrow n_F + 1$  //policy F owns this result
        else
            $A \leftarrow A \cup \{a_G\},\ n_G \leftarrow n_G + 1$  //policy G owns this result
        end if
    end if
end for
return $A$
```

- Another alternative is to show results for twenty days, with five articles per day, and each algorithm running for ten days. Again both algorithms would recommend from the same day for consecutive days. However, preliminary experiments suggest that the number of days causes more user fatigue than showing more articles per day (with respect to our user study). Furthermore, it is unclear what the effect is of repeatedly showing articles from the same day.

After each each day, the user's feedback is collected and given to the two competing algorithms. The policies only receive feedback corresponding to results that they own. The reward for each slot is a binary variable corresponding to whether or not the user liked the article. To avoid stability issues, we allow both policies to observe all feedback during the first two days of each evaluation.

User Interface. Figure 5 shows a snapshot of the decision pane in our user interface (with the preview pane displaying article content located to the right). Articles are shown to the user one by one, and the user is told to either click the check (like) or cross (dislike) depending on whether they are interested in reading the article in detail given the articles already shown that day. The study is designed to take approximately 20 minutes.

Dimensionality Reduction. We performed dimensionality reduction by creating 18 genres, labeled relevant articles and trained a model via linear regression for each genre. We then represent coverage for each genre i as

$$F_i(A) = 1 - \prod_{\ell} \left(1 - \pi_i^\top \phi(a^{(\ell)} | A^{(1:\ell-1)})\right),$$

where π_i denotes the learned linear regression model for genre i, and $\phi(a|A)$ denotes the component-wise probabilistic coverage of the lower-level features (generated using LDA in our setting), i.e.

$$\phi(a|A) = a \left(1 - \prod_{a' \in A} (1 - a')\right).$$

Although this new definition of F_i is no longer a set function (and thus does not enjoy the same theoretical properties), we observe that it performs well in practice. Designing more principled yet effective dimensionality reduction models is an important direction for future work.

Algorithm Parameter Settings. We set $\alpha_t = 1/t$ for LSBGreedy, $\alpha = 0.6/t$ for RankLinUCB, and $\beta = 0.5$ for MW as according to [9]. Because our user study lasts for only 10 iterations, we found that decreasing the emphasis on exploration over time for LSBGreedy and RankLinUCB provided some performance gains. We kept the learning rate for MW constant, since decreasing the learning rate over time did not seem to improve performance. This also allows us to make a more meaningful comparison with previous work [9].