Reinforcement Learning

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What is Reinforcement Learning?

RL: general framework for online decision making given partial and delayed rewards

- learner is an agent that performs actions
- actions influence the state of the environment
- environment returns reward as feedback

Generalization of the Multi-Armed Bandit problem

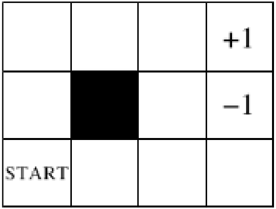
Markov Decision Processes (MDP)

Models the environment that we are trying to learn

Tuple (S, A, P_a, R, γ)

- S the set of states (not necessarily finite)
- A the set of actions (not necessarily finite)
- $P_a(s, s')$ the transition probability kernel
- $R : S \times A \to \mathbb{R}$ the reward function
- $\gamma \in (0, 1)$ the discount factor

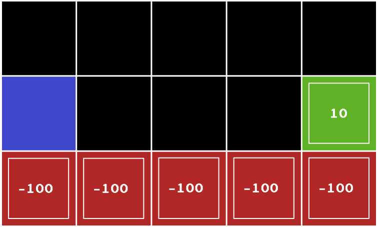
GridWorld MDP Example



- States: each cell of the grid is a state
- Actions: move N, S, E, W, or stationary (can't move off grid or into wall)
- Transitions: Deterministic, move into cell in action direction
- Rewards: 1 or -1 in special spots, 0 otherwise

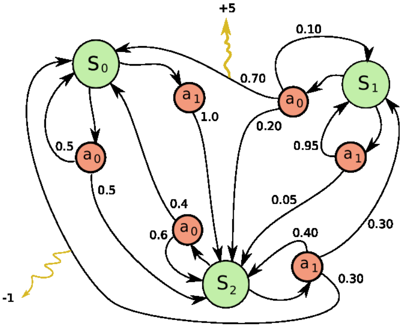
Simulation . . .

Another GridWorld Example



- States: each cell of the grid is a state
- Actions: move N, S, E, W (can't move off grid or into wall)
- Transitions: Deterministic, move into cell in action direction. Any move from 10 or -100 transitions to Start.
- Rewards: 10 or -100 moving out of special spots, 0 otherwise

MDP Overview Example



- Three states $S = \{S_0, S_1, S_2\}$.
- Two actions for each states $A = \{a_0, a_1\}$.
- Probabilistic transitions P_a.
- Rewards defined by $R : S \times A \to \mathbb{R}$.

Markov Property

- Markov Decision Processes (MDP) are very similar to Markov chains. An important property is the **Markov Property**.
- **Markov Property**: Set of possible actions and probability of transitions does not depend on the sequence of events that preceded it. In other words, the system is *memoryless*.
- Sometimes not completely satisfied, but approximation is good enough.

Episodic vs Continuing RL

- Two classes of RL problems:
- Episodic problems are separated by terminations and restarting, such as losing in a game and having to start over.
- Continuing problems are single-episode and continue forever, such as a personalized home assistance robot.

Objective

- Pick the actions that lead to the best future reward
- ”best” $\longleftrightarrow$ maximize expected future discounted return:

$$R_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots = \sum_{t' \geq t} \gamma^{t'-t} r_{t'}$$

- Discount factor $\gamma \in (0, 1)$
  - avoids infinite return
  - encodes uncertainty about future rewards
  - encodes bias towards immediate rewards

Using a discount factor γ is only one way of capturing this.

Policy and Value

- Policy: $\pi : S \to \mathcal{P}(A)$ - given a state, the probability distribution of the action the agent will choose
- Value: $Q^\pi(s_t, a_t) = \mathbb{E}[R_t | s_t, a_t]$ - given some policy π, the expected future reward under some state and action

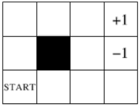
---

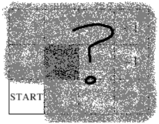
- Compare to the MAB definitions:
  - Policy: Pick an action a_i. For example, UCB1 can be used to determine what action to pick.
  - Value: The expected reward μ_i associated with each action.

RL vs. Bandits

- Reinforcement learning is an extension of bandit problems.
- Standard stochastic MAB problem $\longleftrightarrow$ single-state MDP.
- Contextual bandits can model state, but not transitions
- **Key point:** RL utilizes the entire MDP (S, A, P_a, R, γ). RL can account for delayed rewards and can learn to "traverse" the MDP states.
- No regret analysis for RL (too difficult, hard to generalize). MAB is more constrained, so it is easier to analyze and bound.

Model-based vs. Model-free RL





Model-based approaches assume information about the environment

Do we know the MDP (in particular its transition probabilities)?

- Yes: can solve MDP exactly using dynamic programming/value iteration
- No: try to learn the MDP (e.g. E^3 algorithm[1])

Model-free: learn a policy in absence of a model

- We will focus on model-free approaches

[1] Kearns and Singh (1998)

Model-free approaches

Optimize either value or policy directly - or both!

- Value-based:
  - Optimize value function
  - Policy is implicit
- Policy-based:
  - Optimize policy directly
- Value and policy based:
  - Actor-critic[2]

We will mostly consider value-based approaches.

[2] Konda and Tsitsiklis 2003

Value-based RL

- Define optimal value function to be the best payoff among all possible policies:

$$Q^*(s, a) = \max_{\pi} Q^{\pi}(s, a)$$

  Recall π are the policies and Q^{π} are the value functions.
- Value-based approaches: learn optimal value function
- Simple to derive a target policy from optimal value function

Exploration vs. Exploitation in RL

- Important concept for both RL and MAB
- Relevant in learning stage
- Fundamental tradeoff: agent should explore enough to discover a good policy, but should not sacrifice too much reward in the process
- ϵ-greedy strategy:
  Pick the ‘optimal’ strategy with probability $1 - \epsilon$, and select a random action with probability ϵ.

- Recall that the value function is defined as

$$Q^{\pi}(s_t, a_t) = \mathbb{E}[R_t | s_t, a_t]$$

- Recall that we can solve the RL problem by learning the optimal value function

$$Q^*(s, a) = \max_{\pi} Q^{\pi}(s, a)$$

Bellman equation

- Suppose action a leads to state s'. We can expand the value function recursively:

$$Q^{\pi}(s, a) = \mathbb{E}_{s'}[r + \gamma \max_{a'} Q^{\pi}(s', a')|s, a]$$

- Solve using value iteration:

$$Q^{\pi}_{i+1}(s, a) = \mathbb{E}_{s'}[r + \gamma \max_{a'} Q^{\pi}_{i}(s', a')|s, a]$$

Approximating the expectation

- If we know the MDP's transition probabilities, we can just write out the expectation:

$$Q(s,a) = \sum_{s'} p_{ss'}(r + \gamma \max_{a'} Q(s',a'))$$

- Q-learning approximates this expectation with a single-sample iterative update (like in SGD)

- Iteratively solve for optimal action-value function Q^* using Bellman equation updates

$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha_t[r_t + \gamma \max_{a'} Q(s', a') - Q(s_t, a_t)]$$

  for learning rate α_t

- Intuition for value iteration algorithms: a la gradient descent, iterative updates (hopefully) lead to desired convergence

Target vs. training policy

We distinguish between action selection policies during training and test time.

- Training policy: balance exploration and exploitation such as
  - ϵ-greedy (most commonly used)
  - Softmax

$$\sigma(z_i) = \frac{e^{z_i}}{\sum_{k=1}^{K} e^{z_k}}$$

- Target policy: pick the best possible action (highest Q-value) every time

Q-learning algorithm

```
Init Q(s, a) = 0 ∀(s, a) in S × A
while not converged do
    t += 1
    pick and do action a_t according to current policy (e.g. ε-greedy)
    receive reward r_t
    observe new state s'
    update Q(s_t, a_t) = Q(s_t, a_t) + α_t [r_t + γ max_{a'} Q(s', a') − Q(s_t, a_t)]
end while
```

Init $Q(s,a) = 0 \forall (s,a) in S \times A$
while not converged do
  $t += 1$
  pick and do action a_t according to current policy (e.g. ϵ-greedy)
  receive reward r_t
  observe new state s'
  update $Q(s_t, a_t) = Q(s_t, a_t) + \alpha_t [r_t + \gamma \max_{a'} Q(s', a') - Q(s_t, a_t)]$
end while

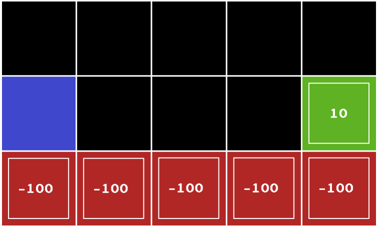
On-policy vs. off-policy algorithm

- Q-learning is an **off-policy** algorithm
    - learned Q function approximates Q^* independent of policy being used
- **On-policy** algorithms perform updates that depend on the policy, such as SARSA:

$$Q(s_t, a_t) = (1 - \alpha)Q(s_t, a_t) + \alpha_t[r_t + \gamma Q(s_{t+1}, a_{t+1})]$$

- Convergence properties dependent on policy

Q-learning GridWorld Example



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Q-learning GridWorld Details

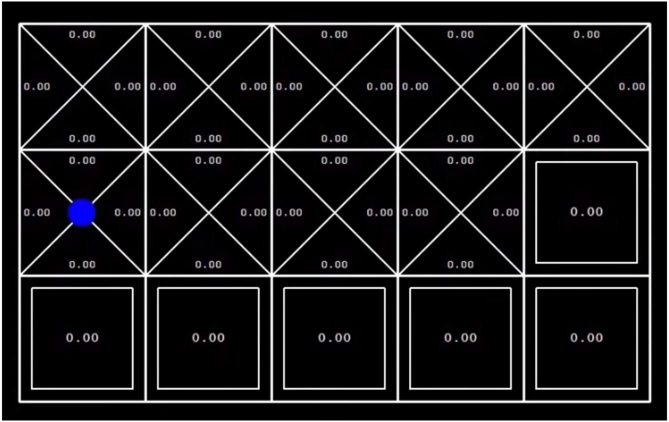
Recall Bellman equation update

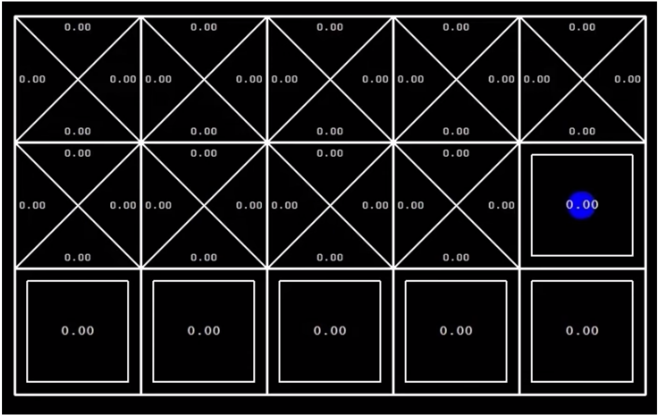
$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha_t [r_t + \gamma \max_{a'} Q(s', a') - Q(s_t, a_t)]$$

We have

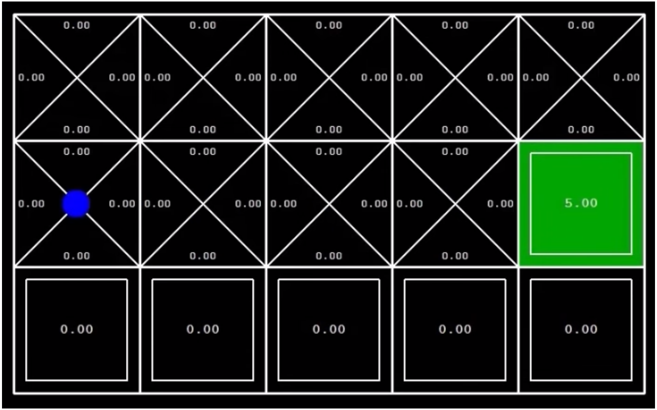
- $\alpha = 0.5$ (for fast updates; usually much smaller)
- $\gamma = 1$

Walkthrough: Initial state



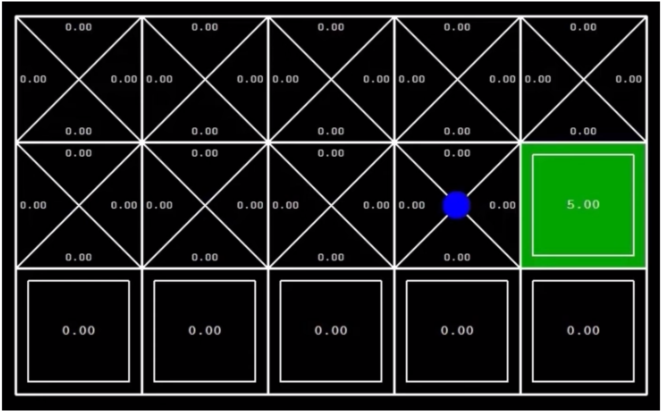


Let’s say the agent keeps on moving right until he reaches the exit

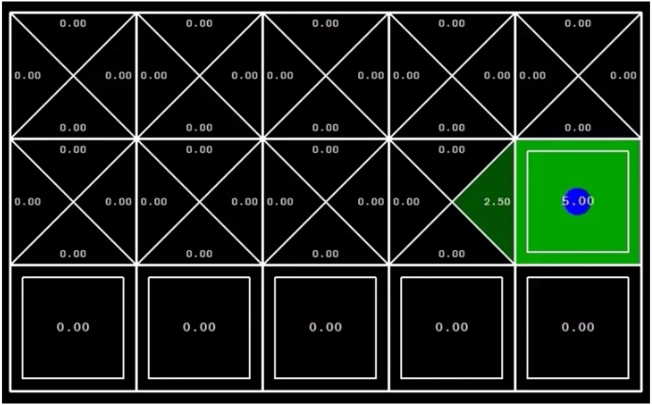


$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha_t[r_t + \gamma \max_{a'} Q(s', a') - Q(s_t, a_t)]$$

$$Q(s^*, a) = 0 + 0.5[10 + 0 - 0] = 5$$

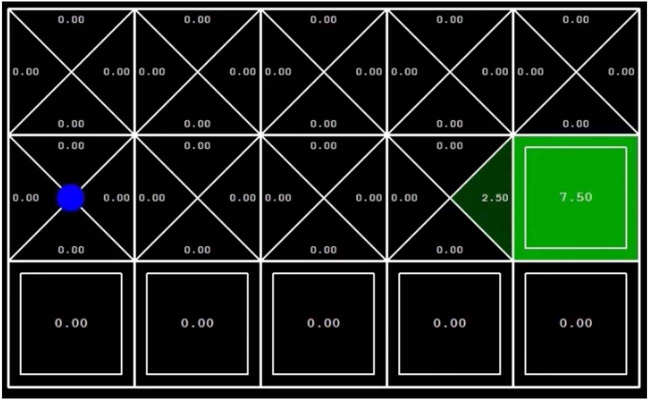


What happens if we reach the exit again?



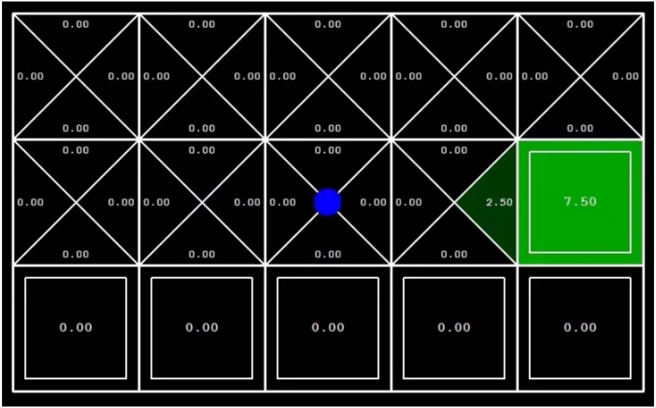
$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha_t[r_t + \gamma \max_{a'} Q(s', a') - Q(s_t, a_t)]$$

$$Q(s, a = E) = 0 + 0.5[0 + 5 - 0] = 2.5$$



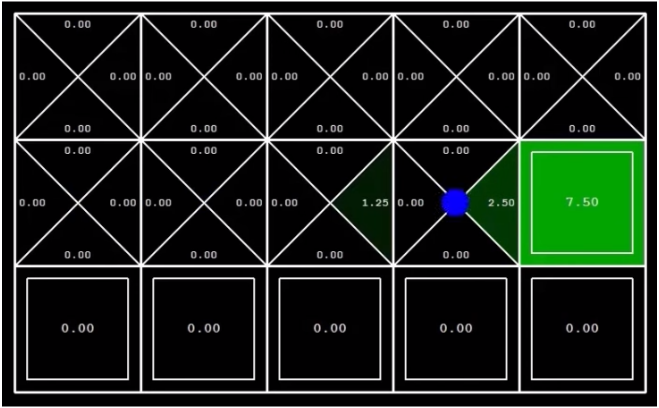
$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha_t [r_t + \gamma \max_{a'} Q(s', a') - Q(s_t, a_t)]$$

$$Q(s, a = E) = 5 + 0.5[10 + 0 - 5] = 7.5$$



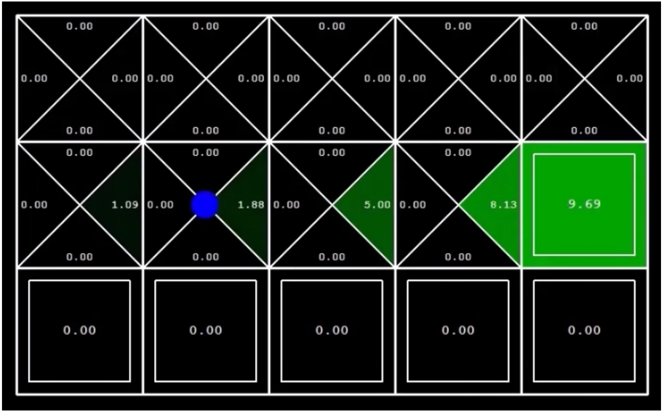
What happens if we keep on going east?

$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha_t[r_t + \gamma \max_{a'} Q(s', a') - Q(s_t, a_t)]$$

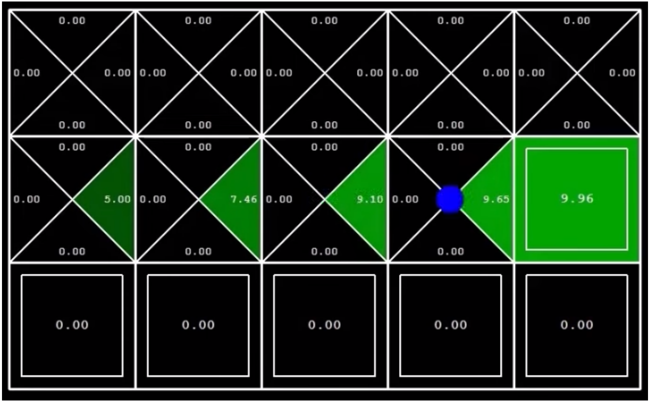


$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha_t [r_t + \gamma \max_{a'} Q(s', a') - Q(s_t, a_t)]$$

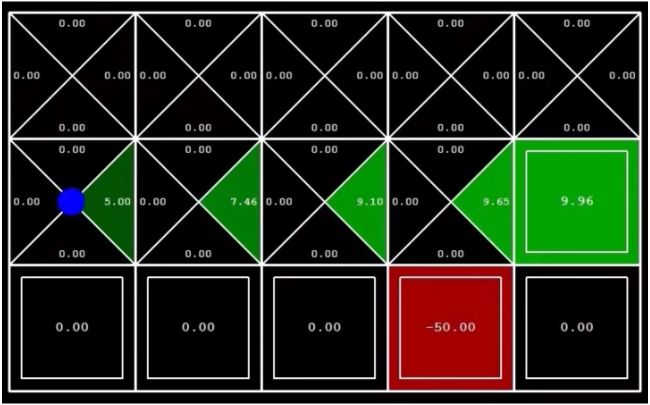
$$Q(s, a = E) = 0 + 0.5[0 + 2.5 - 0] = 1.25$$



After going only east for several episodes

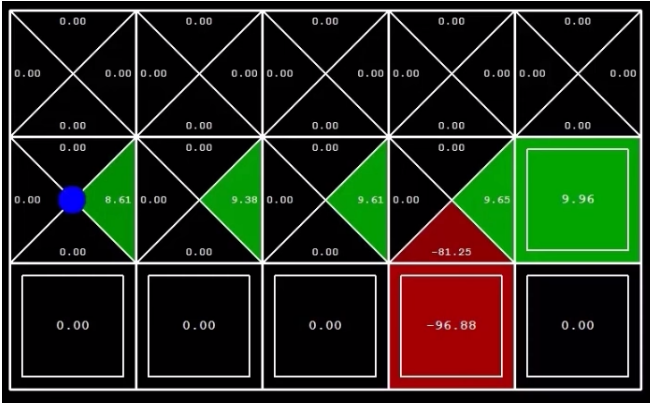


What if we go south?



$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha_t [r_t + \gamma \max_{a'} Q(s', a') - Q(s_t, a_t)]$$

$$Q(s, a) = 0 + 0.5[-100 + 0 - 0] = -50$$


0.00
0.00
0.00
0.00
8.61
9.38
9.61
9.65
9.96
-81.25
-96.88

Recall that update is greedily optimistic:

$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha_t[r_t + \gamma \max_{a'} Q(s', a') - Q(s_t, a_t)]$$

Q-learning Convergence

Two major assumptions:

i. Every state is visited infinitely often

ii. Learning rate α_t satisfies

$$\sum_{t=1}^{\infty} \alpha_t = \infty \qquad \sum_{t=1}^{\infty} \alpha_t^2 < \infty$$

Theorem

Q-learning converges to the optimal action-value function $Q^*(s,a)$ with probability 1 given i. and ii.

Proof: use stochastic approximation ideas.

Proof Sketch

Lemma

A random iterative process $\Delta_{t+1}(x) = (1 - \alpha_t(x))\Delta_t(x) + \alpha_t(x)F_t(x)$ *convergences to zero w.p.1 under the following assumptions:*

i. $\sum_{t=1}^{\infty} \alpha_t = \infty \qquad \sum_{t=1}^{\infty} \alpha_t^2 < \infty$

ii. $||\mathbb{E}[F_t(x)|\mathcal{F}_t]||_W \leq \gamma ||\Delta_t||_W$ *for* $\gamma \in (0, 1)$

iii. $Var[F_t(x)|\mathcal{F}_t] \leq C(1 + ||\Delta_t||_W^2)$ *for some constant* C

- x denotes state.
- drop dependence on state for clarity
- $\|\cdot\|_W$ denotes some weighted max norm - can just analyze for sup norm

Applying the lemma

Rewrite Bellman equation update:

$$Q_{t+1}(s_t, a_t) = (1 - \alpha_t)Q_t(s_t, a_t) + \alpha_t(r_t + \gamma \max_{a'} Q_t(s_{t+1}, a'))$$

Subtract $Q^*(s_t, a_t)$ from both sides:

$$\begin{aligned} Q_{t+1}(s_t, a_t) - Q^*(s_t, a_t) &= (1 - \alpha_t)(Q_t(s_t, a_t) - Q^*(s_t, a_t)) \\ &\quad + \alpha_t(r_t + \gamma \max_{a'} Q_t(s_{t+1}, a') - Q^*(s_t, a_t)) \\ \Delta_{t+1} &= (1 - \alpha_t)\Delta_t + \alpha_t F_t \end{aligned}$$

Proof boils doing to showing that requirements 2 and 3 of the lemma are satisfied

- First follows from fact that value iteration update F_t is a contraction mapping.
- Second follows by expanding and noting that rewards are bounded.
- See [2] for details.

Function Approximation

- Vanilla Q-learning for finite MDPs stores values in a lookup table
- Obviously intractable for large or continuous MDPs
- However, we can replace this with a function approximator
- Find some model Q with parameters θ s.t.

$$Q(s, a, \theta) \approx Q^*(s, a)$$

  - Linear models
  - Gaussian processes
  - Neural networks

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Deep Q-Learning

- Approximates the value function using a deep network.
  - Non-linear function approximator
- Approximate the value function $Q(s, a, w) \approx Q^{\pi}(s, a)$
- Objective function is mean-squared error of Q-values

$$\mathcal{L}(w) = \mathbb{E}\left[\left(r + \gamma_{a'} Q(s', a', w) - Q(s, a, w)\right)^2\right]$$

- Train using gradient descent

$$\nabla \mathcal{L} = \mathbb{E}\left[\left(r + \gamma_{a'} Q(s', a', w) - Q(s, a, w)\right) \nabla Q(s, a, w)\right]$$

Atari

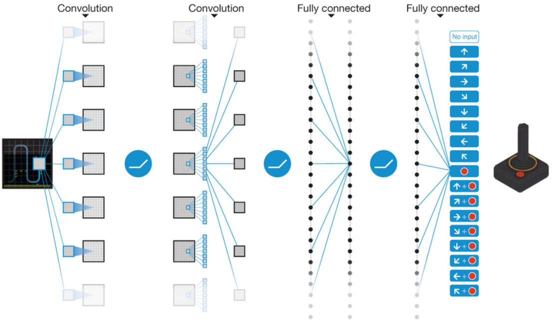
Arcade Learning Environment (ALE): pixel-level games

- Receive as input a 210x160 image with 128 colors and current score
- Action is any of the 18 buttons/joy stick movements
  - Actions unlabeled (ie no specification for up button)
- Still largely unsolved (even after DQN!)

Main challenges:

- Input is very high-dimensional (vision in the form of pixels)
- Long-term planning is difficult (delay between action and reward)

Convolutional Neural Networks

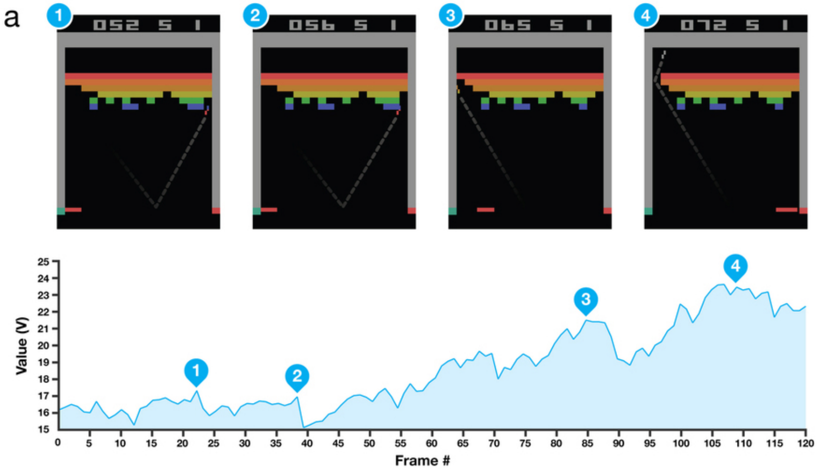


- Convolutional filters mirror the way we see
  - Same filter applies through sliding window across image
  - substantially decreases number of weights needed
- Subsampling of results
  - Take average or max of sliding window
  - translational invariance
- End with fully connected layers

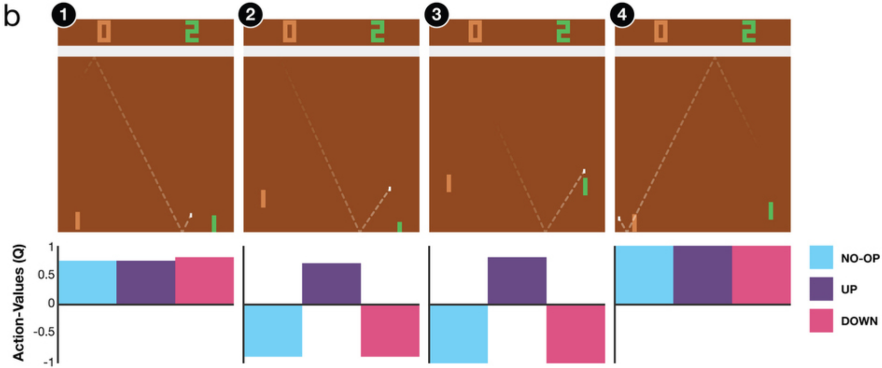
Preprocessing

- CNN on raw CMYK data
  - Pre-processed images by downscaling from 210x160 to 110x84 then cropping to 84x84
  - Max of two frames used to account for flickering
  - Extracted solely Y (luminance) channel
  - Final fully-connected layer to separate output units for each action
- Action selected every k frames for faster training

Q-network Example



Q-network Example



Atari-specific problems

Training deep RL networks directly leads to bad performance

- Adjacent training samples are clearly correlated
- Break correlations
  - experience replay
- Unstable gradients from unknown reward scale
  - clip rewards
- Oscillation from policy and Q-network changing
  - Fix Q-network

Experience Replay

- Build dataset from agent's own experience
  - Store last N transitions $(s_t, a_t, r_{t+1}, s_{t+1})$ in replay memory $\mathcal{D}$
  - At each iteration, sample random mini-batch $U(\mathcal{D})$ of transitions from $\mathcal{D}$
  - Recall Bellman equation $Q(s,a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q(s', a') | s, a\right]$
  - Target $y = r + \gamma \max_{a'} Q(s', a', w)$

$$\mathcal{L}(w) = \mathbb{E}_{(s,a,r,s') \sim U(\mathcal{D})}\left[(y - Q(s,a,w))^2\right]$$

$$\nabla_w = \mathbb{E}_{s,a,r,s'}\left[\left(r + \gamma \max_{a'} Q(s', a', w] - Q(s,a,w)\right) \nabla_w Q(s,a,w)\right]$$

Reward clipping

- Clip rewards to $\{-1, 1\}$
  - Keeps Q-values small
  - Can use same gradient descent parameters
  - Can't tell difference between small and large rewards

Q-network Stability

- Fix Q-network every C updates to a target network $\hat{Q}$
  - Denote saved weights $\hat{w}$
- Use $\hat{Q}$ to generate Q-learning targets y
- Less likely to have oscillations between y and Q changes

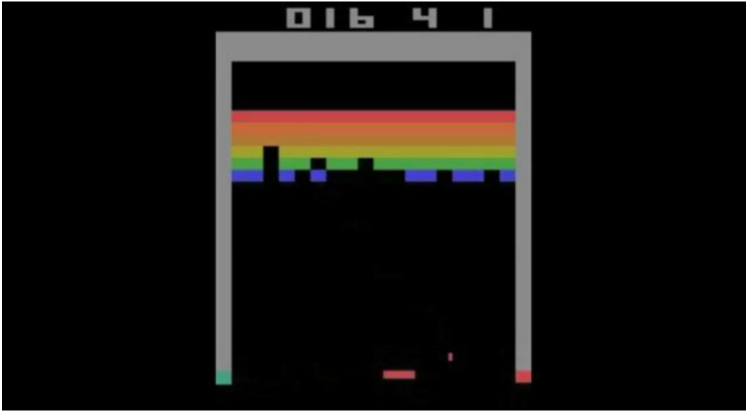
$$\nabla_w = \mathbb{E}_{s,a,r,s'}\left[\left(r + \gamma \max_{a'} Q(s', a', \hat{w}] - Q(s, a, w)\right) \nabla_w Q(s, a, w)\right]$$

initialize replay memory $\mathcal{D}$
initialize action-value Q randomly
for episode = $1, M$ **do**
  initialize sequence s_1 and preprocessed sequence ϕ_1
  for $t = 1, T$
  select random action a_t with probability ϵ
  else select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$ **do**
    execute action a_t in emulator and observe reward r_t and image x_{t+1}
    store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
    sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
    set $y_j = r_j$ for terminal ϕ_{j+1} and
    $y_j = r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta)$ for non-terminal ϕ_{j+1}
    perform gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$
  end for
end for

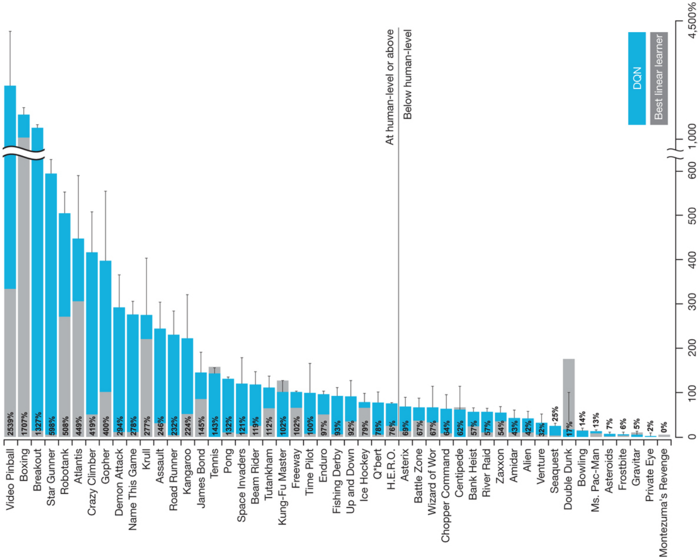
Example

Water World

Example



DQN results


DQN
Best linear learner
At human-level or above
Below human-level
0
100
200
300
400
500
600
1,000
4,500%
Video Pinball 2539%
Boxing 1707%
Breakout 1327%
Star Gunner 598%
Robotank 508%
Atlantis 449%
Crazy Climber 419%
Gopher 400%
Demon Attack 294%
Name This Game 278%
Krull 277%
Assault 246%
Road Runner 232%
Kangaroo 224%
James Bond 145%
Tennis 143%
Pong 132%
Space Invaders 121%
Beam Rider 119%
Tutankham 112%
Kung-Fu Master 102%
Freeway 102%
Time Pilot 100%
Enduro 97%
Fishing Derby 93%
Up and Down 92%
Ice Hockey 79%
Q*bert 78%
H.E.R.O. 76%
Asterix 69%
Battle Zone 67%
Wizard of Wor 67%
Chopper Command 64%
Centipede 62%
Bank Heist 57%
River Raid 57%
Zaxxon 54%
Amidar 43%
Alien 42%
Venture 32%
Seaquest 25%
Double Dunk 17%
Bowling 14%
Ms. Pac-Man 13%
Asteroids 7%
Frostbite 6%
Gravitar 5%
Private Eye 2%
Montezuma's Revenge 0%

Long-term Planning

- Performs poorly in games requiring long-term planning
- Low probability of finding exact sequence of events with $\epsilon -$ greedy
  - Sequence of *n* exact events is found with probability exponential to *n*
- Q-network has no memory state
- DQRN tries to remedy this with LSTM layer replacing fully connected layer
  - Partially successful on long term games

Breakout trained for 24 hours on Titan X

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