Caltech

Machine Learning & Data Mining

CS/CNS/EE 155

Lecture 15:

Deep Learning

Announcements

- Miniproject 2 Released
  - Poem Generation using HMMs
  - Due March 10th
- Final Exam will be released on March 14^{th}
  - Take-home (via Moodle)
  - Intended to take 3 hours (shorter than homeworks)
  - 24-36 hour window
  - Open book of everything on course website
  - No collaboration

Recap: Linear Models

- Linear scoring function in input features:

$$f(x \mid w, b) = w^T x - b$$

- Sometimes non-linear transform at the end
  - E.g., logistic Regression

$$P(y=1 \mid x, w, b) = \tau(f(x \mid w, b)) = \frac{1}{1+\exp\left\{-f(x \mid w, b)\right\}}$$

Recap: Multiclass Logistic Regression

Binary LR: $$P(y=1\mid x,w,b)=\frac{1}{1+e^{-(w^Tx-b)}}$$ $y\in\{0,1\}$

"Log Linear" Property: $$P(y=1\mid x,w,b)\propto e^{w^Tx-b}$$

Extension to Multiclass: $$P(y=k\mid x,w,b)\propto e^{w_k^Tx-b_k}$$

Keep a (w_k,b_k) for each class

Multiclass LR: $$P(y=k\mid x,w,b)=\frac{e^{w_k^Tx-b_k}}{\sum_m e^{w_m^Tx-b_m}}$$ $y\in\{1,\dots,K\}$

Train via Gradient Descent: $$\partial_w\sum_{(x,y)\in S}-\log P(y\mid x,w,b)$$

Example: Handwritten Digit Recognition

$P(y = 2 \mid$  $, w, b)$

$P(y = 9 \mid$  $, w, b)$

- What is feature representation x?
  - – Each pixel is a feature
  - – Logistic regression yields ≈80% accuracy
    - Can we do better?

Errors In Linear Logistic Regression

- Often makes mistakes on 8's:

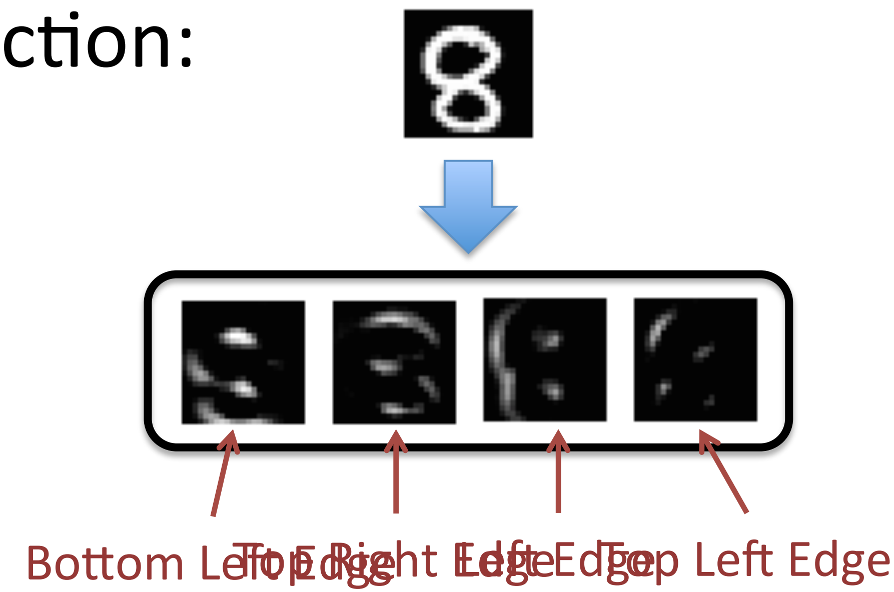
- Shares many pixels with 5's and 3's:

- Linear model on pixels not powerful enough
  - E.g., doesn't capture interactions between pixels

http://cs.stanford.edu/people/karpathy/convnetjs/demo/mnist.html

Feature Engineering

- Linear models require good features x
- Directed edge detection:
  - (With some blurring)
  - “Oriented Gradients”

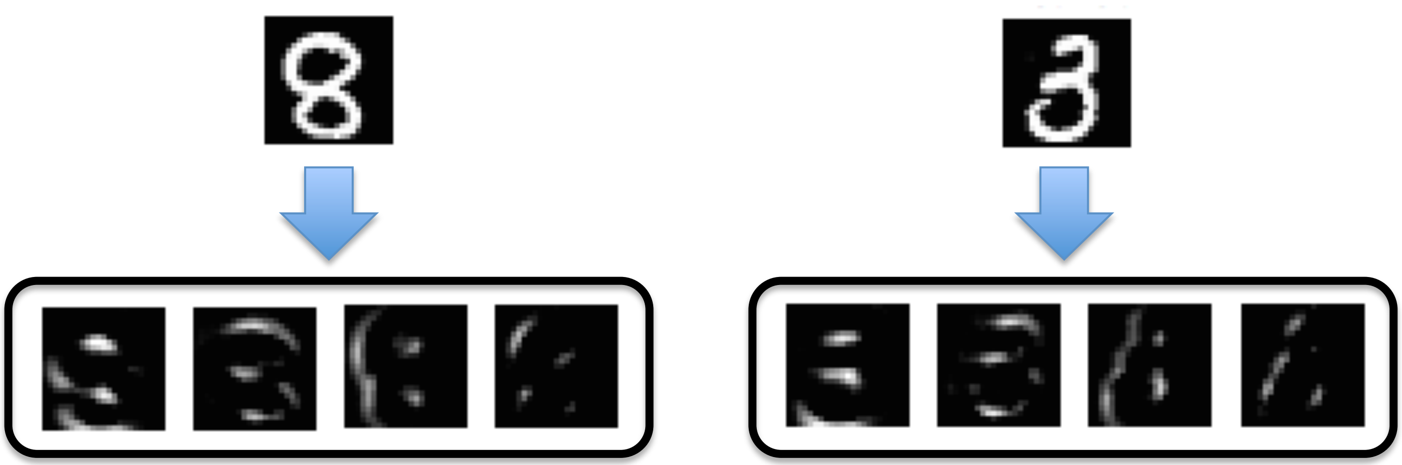


- Logistic regression yields ≈90% accuracy

http://cs.stanford.edu/people/karpathy/convnetjs/demo/mnist.html

Comparing 8's vs 3's

- New feature representation better distinguishes between 8's and 3's:

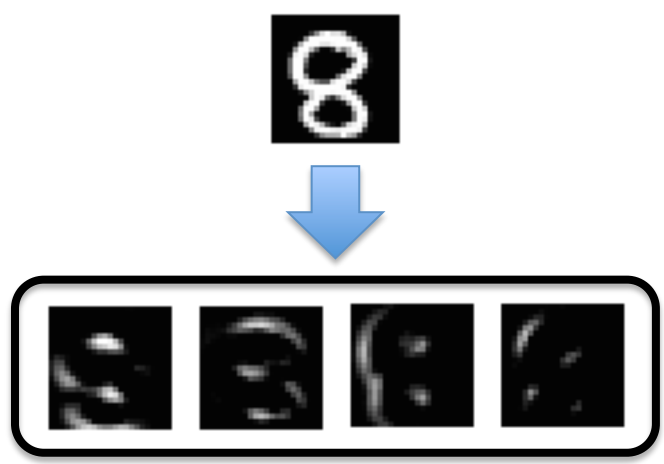


http://cs.stanford.edu/people/karpathy/convnetjs/demo/mnist.html

Learn Features Automatically?

- Feature engineering is tedious
  - Don't know which ones are good
  - Can we just learn them automatically?

- Actually, we did!
  - From convolutional net
    - Learns features
    - Learns logistic regression
    - "Deep Learning"



http://cs.stanford.edu/people/karpathy/convnetjs/demo/mnist.html

Outline For Today

- Introduction to Deep Learning
  - Learning Features for Predictive Modeling
- Deep Convolutional Networks
  - Very popular in Computer Vision
- Tips for Training Deep Networks
- Brief Overview of other Deep Networks

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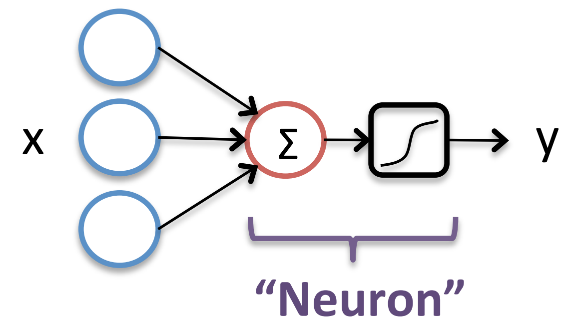
Recap: 1 Layer Neural Network

- 1 Neuron
  - Takes input x
  - Outputs y

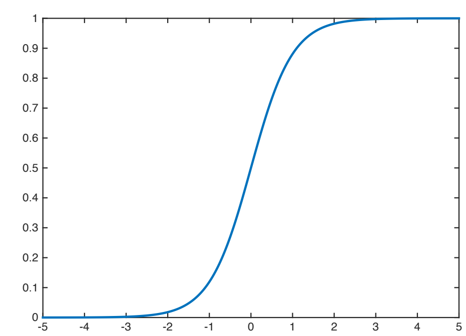


$f(x|w,b) = w^T x - b$

$= w_1 * x_1 + w_2 * x_2 + w_3 * x_3 - b$

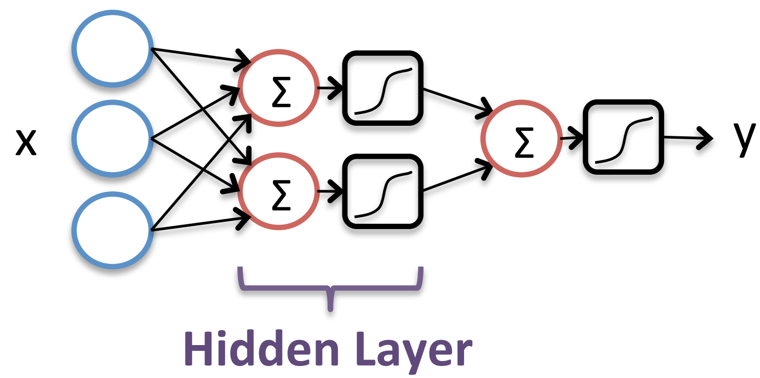
$\longrightarrow y = \tau(\ f(x)\ )$

- **~Logistic Regression!**
  - Gradient Descent



sigmoid
tanh
rectilinear

Recap: 2 Layer Neural Network

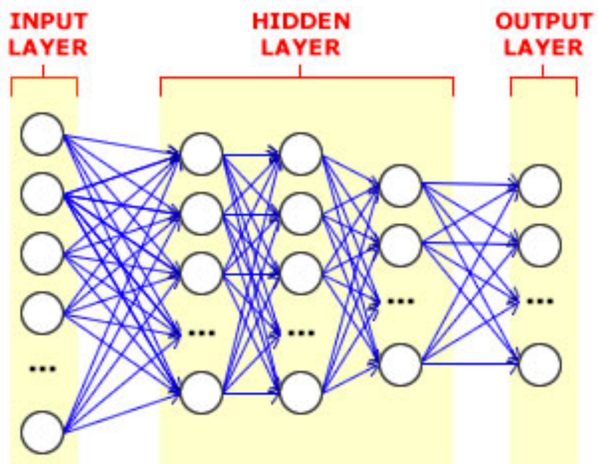


- 2 Layers of Neurons
  - 1^{st} Layer takes input x
  - 2^{nd} Layer takes output of 1^{st} layer

Non-Linear!

- Can approximate arbitrary functions
  - Provided hidden layer is large enough
  - "fat" 2-Layer Network

Deep Neural Networks



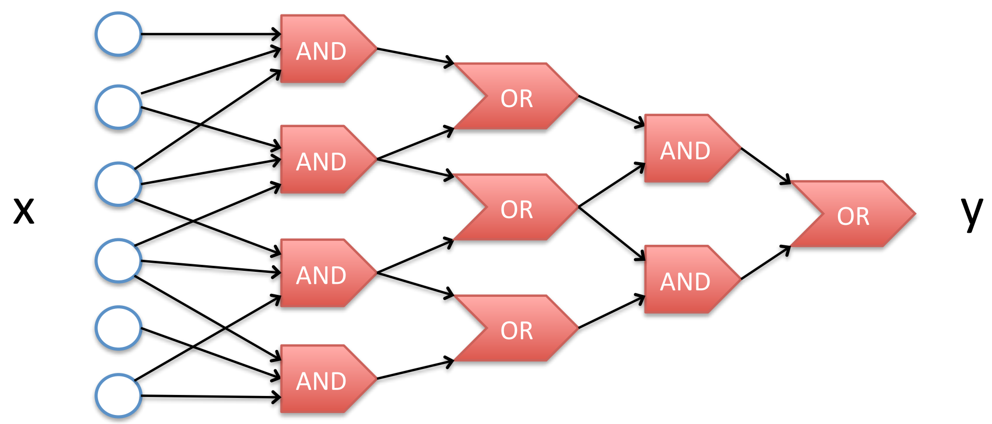
- Why prefer Deep over a "Fat" 2-Layer?
  - Compact Model
    - (exponentially large "fat" model)

Image Source: http://blog.peltarion.com/2014/06/22/deep-learning-and-deep-neural-networks-in-synapse/

Expressive Power

- Deeper networks are "exponentially more expressive" than shallower networks.

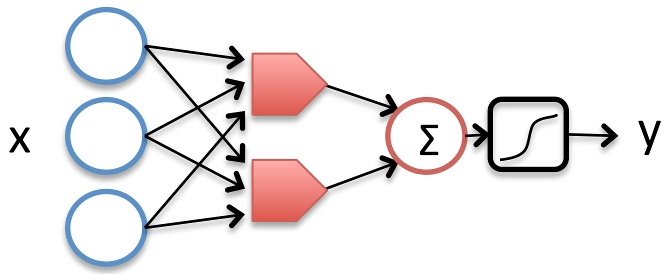
- Related Example: Boolean Circuits
  - **Thought Experiment:** How many gates required if only depth-2 circuits allowed?



http://en.wikipedia.org/wiki/Circuit_complexity

AND & OR as Transfer Functions

- Deep networks can implement AND & OR transfer functions.



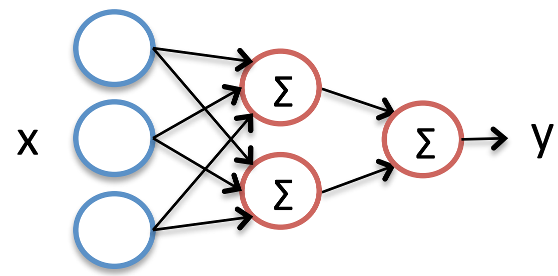
$$\mathrm{AND}\left(f_1, f_2\right) = \min\left\{f_1, f_2\right\}$$

$$\mathrm{OR}\left(f_1, f_2\right) = \max\left\{f_1, f_2\right\}$$

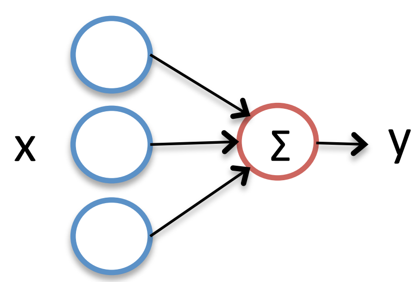
Used in practice

What Happens if No Transfer Function?

- Just linear transforms?

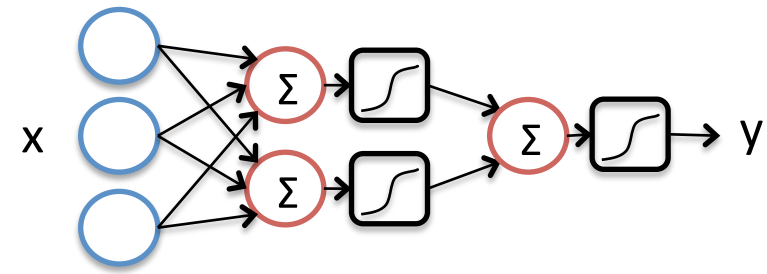


- Deep structure collapses to linear model!
  - Linear operators are associative & commutative
  - Applying a linear operator to a linear operator yields a linear operator



Recap: Training Neural Networks

- Gradient Descent! **
  - (Supervised Learning)



- Parameters:
  - $(w_{11}, b_{11}, w_{12}, b_{12}, w_2, b_2)$

$f(x|w,b) = w^T x - b \quad y = \tau(\, f(x)\,)$

$$\partial_{w_2} \sum_{i=1}^{N} L(y_i, \tau_2) = \sum_{i=1}^{N} \partial_{w_2} L(y_i, \tau_2) = \sum_{i=1}^{N} \partial_{\tau_2} L(y_i, \tau_2) \partial_{w_2} \tau_2 = \sum_{i=1}^{N} \partial_{\tau_2} L(y_i, \tau_2) \partial_{f_2} \tau_2 \partial_{w_2} f_2$$

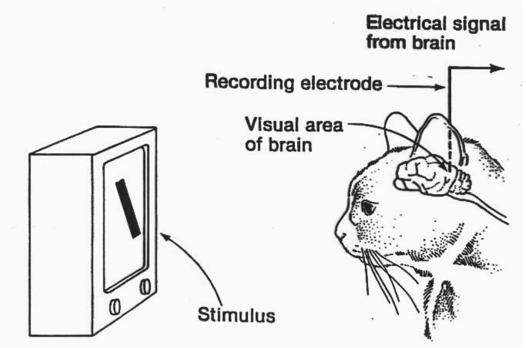
$$\partial_{w_{1m}} \sum_{i=1}^{N} L(y_i, \tau_2) = \sum_{i=1}^{N} \partial_{\tau_2} L(y_i, \tau_2) \partial_{f_2} \tau_2 \partial_{w_{1m}} f_2 = \sum_{i=1}^{N} \partial_{\tau_2} L(y_i, \tau_2) \partial_{f_2} \tau_2 \partial_{\tau_{1m}} f_2 \partial_{f_{1m}} \tau_{1m} \partial_{w_{1m}} f_{1m}$$

Backpropagation = Gradient Descent (lots of chain rules)

**additional details end of lecture

Original Biological Inspiration

- David Hubel & Torsten Wiesel discovered **"simple cells"** and **"complex cells"** in the 1959
  - Some cells activate for **simple patterns**
    - E.g., lines at certain angles
  - Some cells activate for more **complex patterns**
    - Appear to take activations of simple cells as input



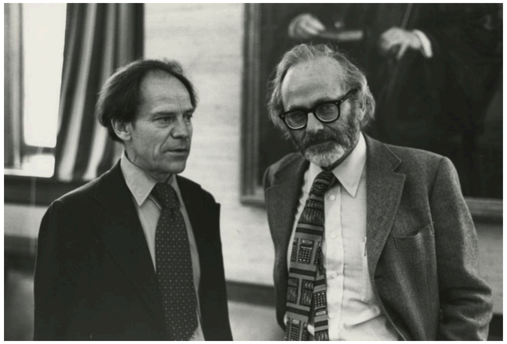
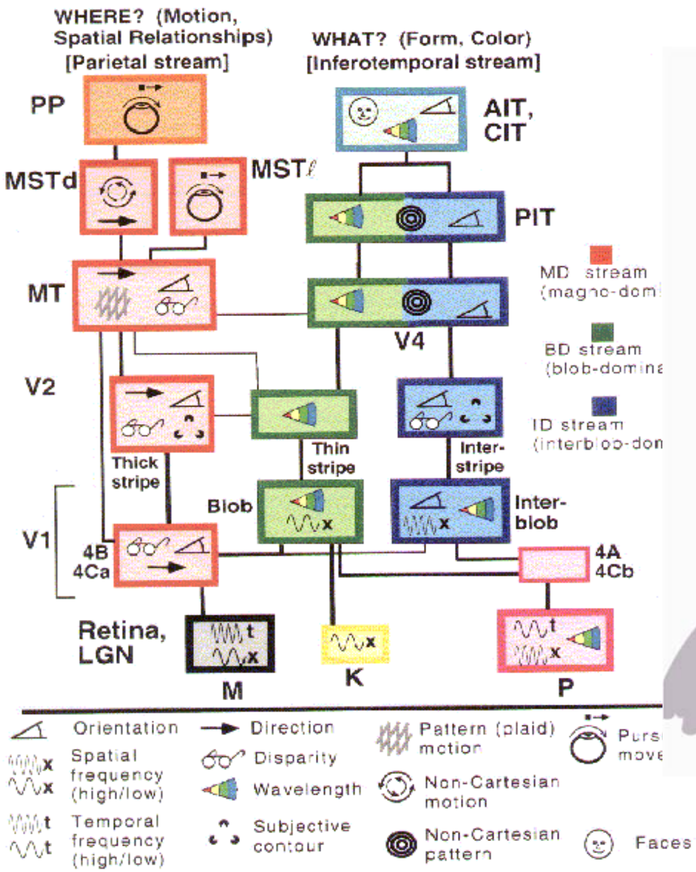
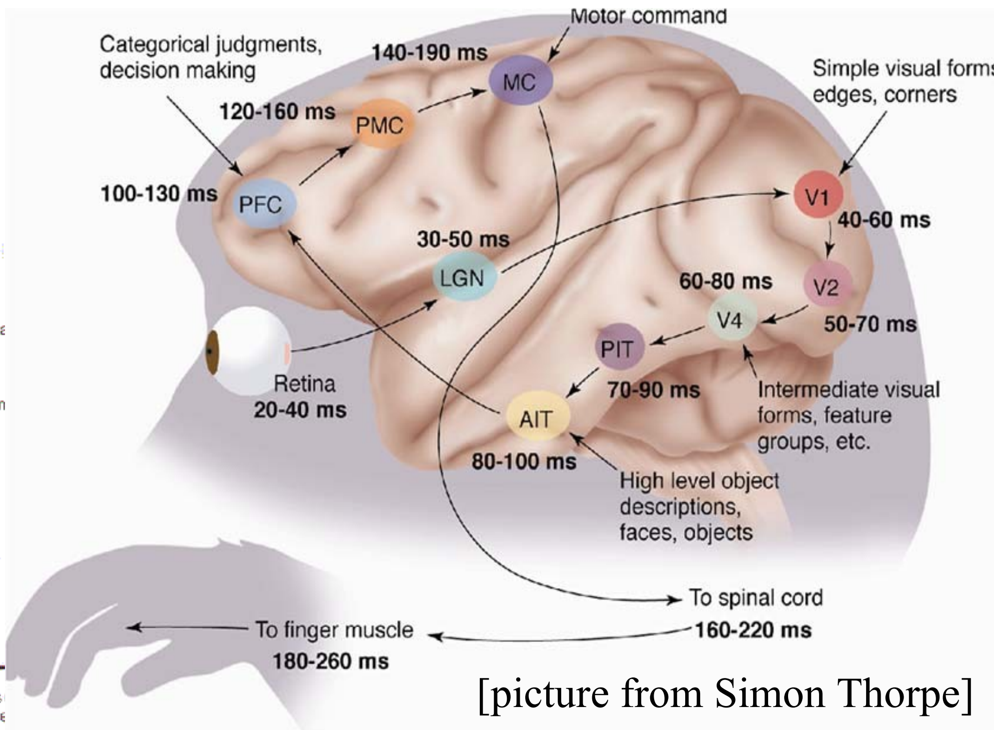


Image Source:
https://cms.www.countway.harvard.edu/wp/wp-content/uploads/2013/09/0002595_ref.jpg
https://cognitiveconsonance.files.wordpress.com/2013/05/c_fig5.jpg

The Brain is Hierarchical





[Gallant & Van Essen]

Image Source: http://www.cs.nyu.edu/~yann/talks/lecun-ranzato-icml2013.pdf

But Let's Not Get Carried Away

- We need some kind of wings to fly
  - But no flapping

- Do we even need wings?

No Longer Biologically Inspired
(for the most part)

- Original inspiration created the feed-forward network

- Field is now called "Deep Learning"
  - Most common name

- Really just Automated Feature Learning
  - Lots of optimization tricks
  - And architecture tuning ← E.g., Convolutional Network

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- Introduction to Deep Learning
  - Learning Features for Predictive Modeling
- **Deep Convolutional Networks**
  - **Very popular in Computer Vision**
- Tips for Training Deep Networks
- Brief Overview of other Deep Networks

Convolutions

- Images typically have invariant patterns
  - E.g., directional gradients are translational invariant:

- Apply convolution to local sliding windows

http://cs.stanford.edu/people/karpathy/convnetjs/demo/mnist.html

Convolutional Filters

- Applies to an image patch x
  - Converts local window into single value
  - Slide across image

$$x \otimes W = \sum_{ij} W_{ij} x_{ij}$$

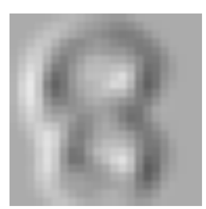
Local Image Patch

Left-to-Right
Edge Detector

-1	0	+1
-1	0	+1
-1	0	+1

W

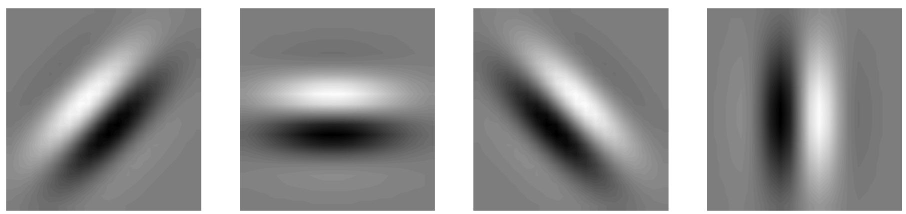
$$\otimes W =$$



Gabor Filters

- Most common low-level convolutions for computer vision

Example W:



- Grey = 0
- Light = positive
- Dark = negative

-1	0	+1
-1	0	+1
-1	0	+1

W

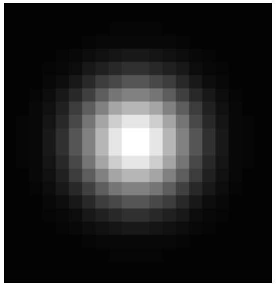
http://en.wikipedia.org/wiki/Gabor_filter

Gaussian Blur Filters

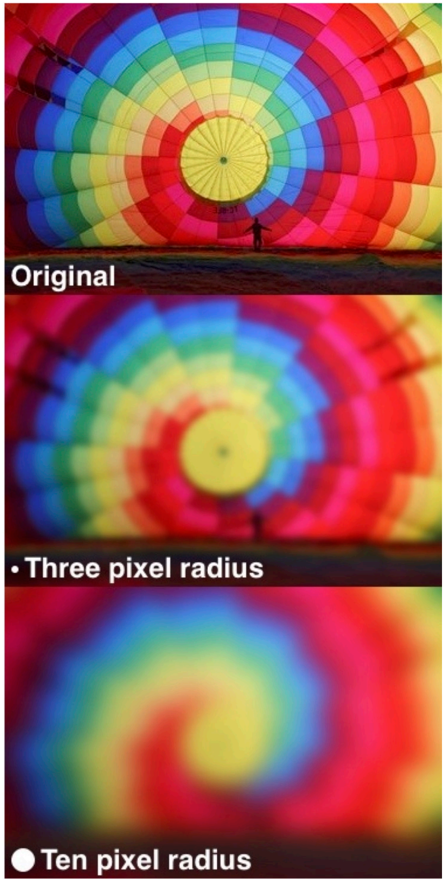
- Weights decay according to Gaussian Distribution
  - Variance term controls radius

Example W:

Apply per RGB Channel



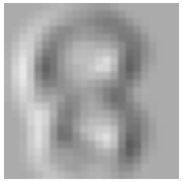
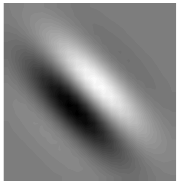
- Black = 0
- White = Positive



http://en.wikipedia.org/wiki/Gaussian_blur

Deep Convolutional Networks

- Learn layers of convolutional filters W
  - Apply convolution to outputs of previous layer

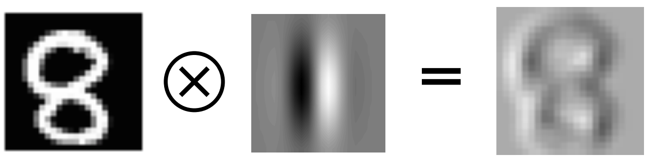
E.g.:  $\otimes$ 

- **Note:** convolutions are linear operators
  - Need non-linear transform
  - Otherwise all layers collapse to single convolution

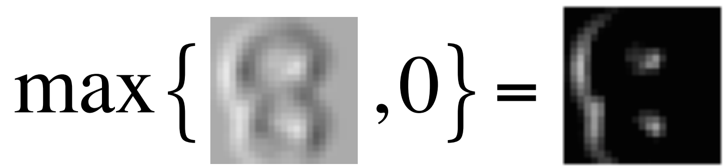
Image Source: http://cs.nyu.edu/~fergus/presentations/nips2013_final.pdf

Convolutional Layer

- Current Convolutional Layer consists of:



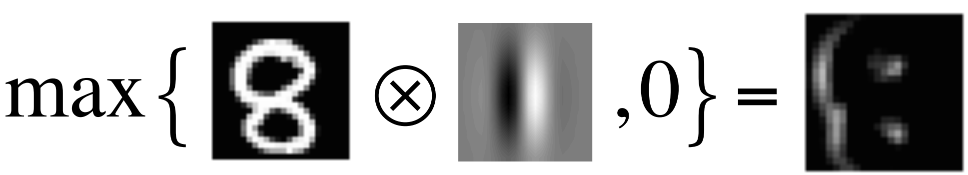
Convolution



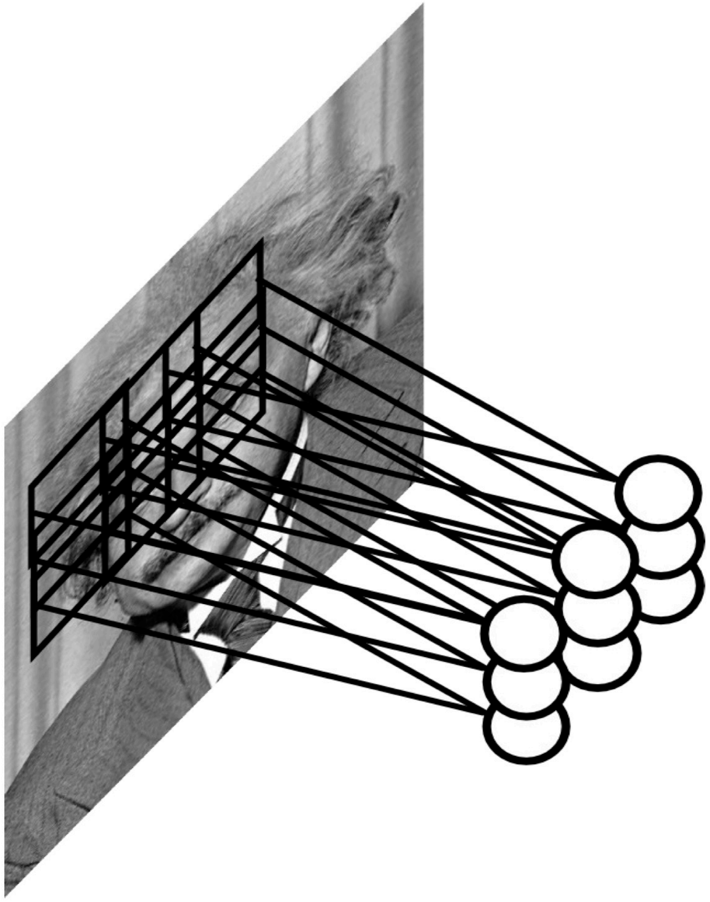
Rectilinear Transform

- Simplifies Backprop
- Chain rule super easy
- Also easier to train

- Main modeling concepts!
  - Combine them to create convolutional layer



Max Pooling

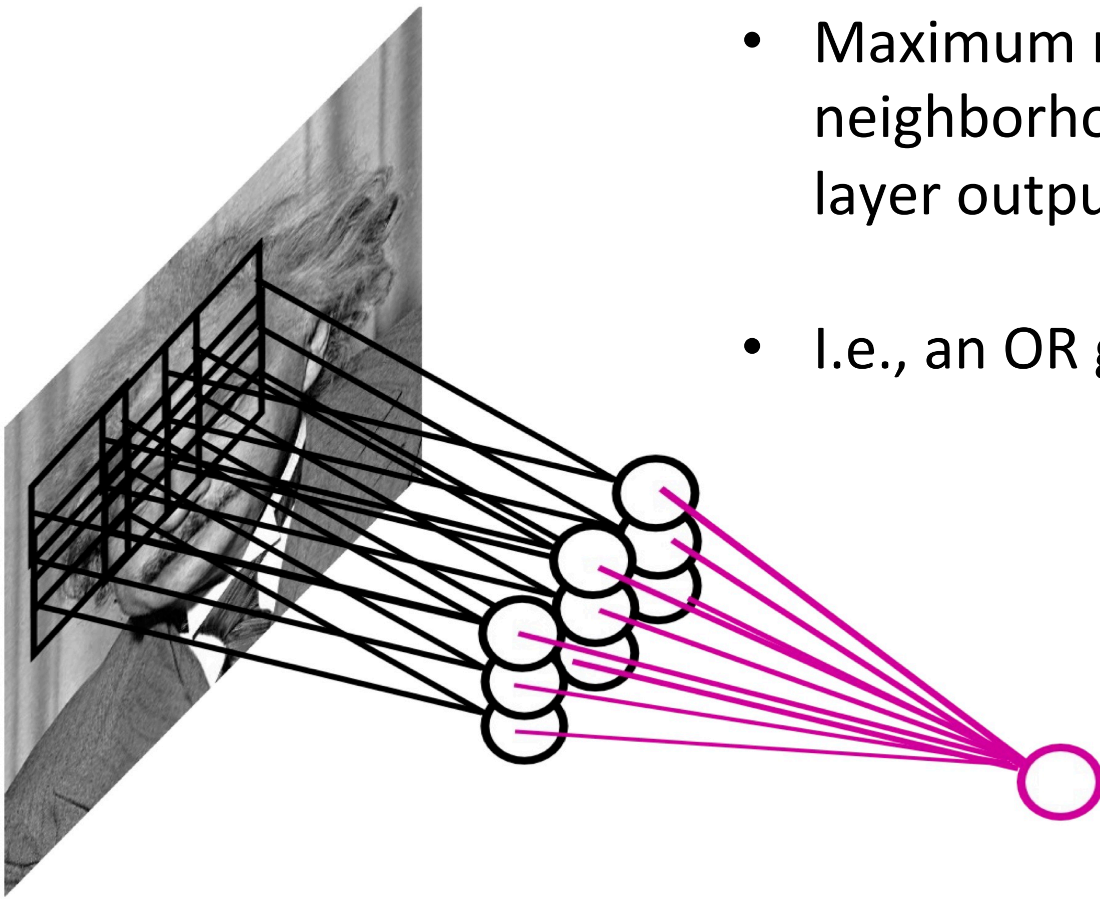


- Assume Convolution Layer is eye detector

- How to make detector more robust to the exact location of the eye?

http://cs.nyu.edu/~fergus/tutorials/deep_learning_cvpr12/tutorial_p2_nnets_ranzato_short.pdf

Max Pooling

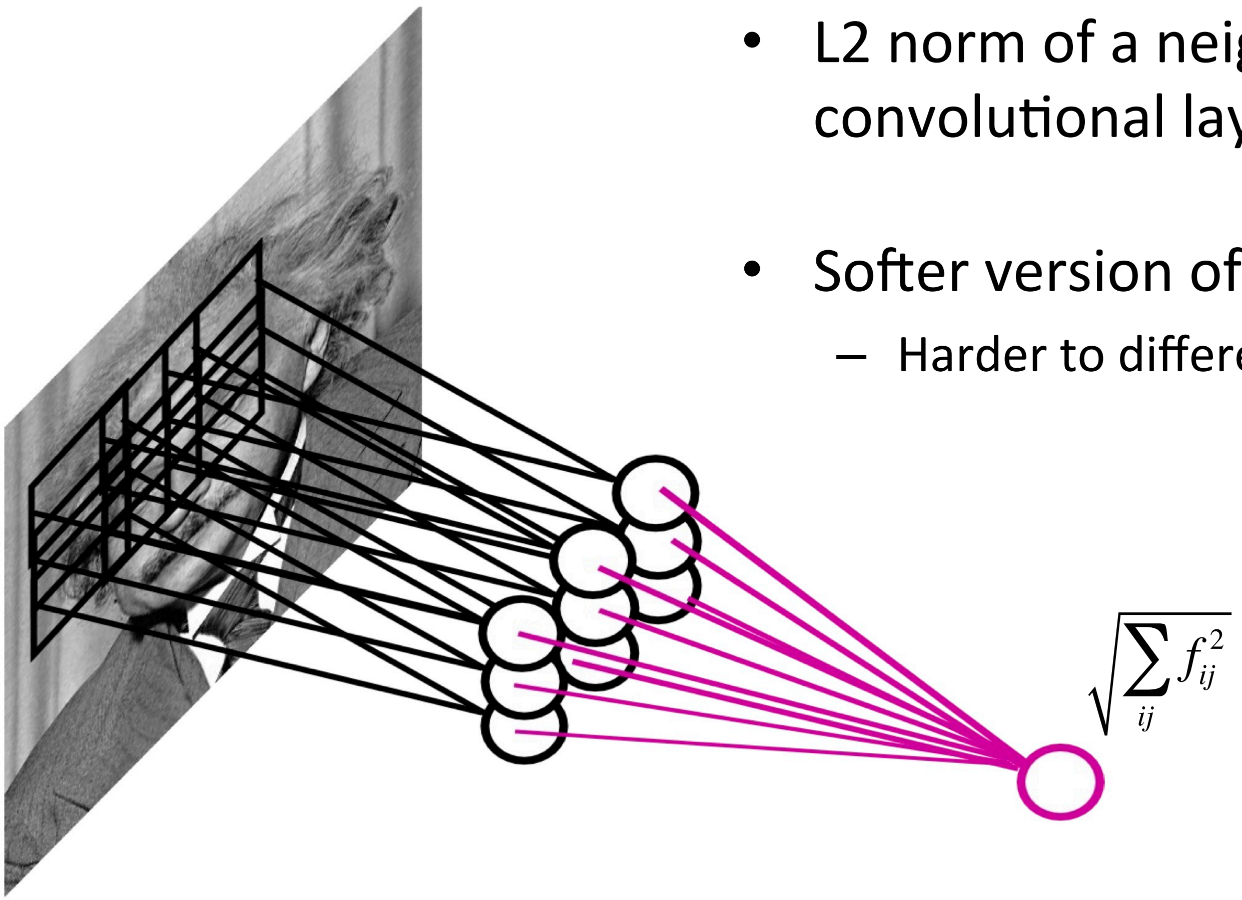


- Maximum response from a neighborhood of convolutional layer outputs
- I.e., an OR gate!

http://cs.nyu.edu/~fergus/tutorials/deep_learning_cvpr12/tutorial_p2_nnets_ranzato_short.pdf

Alternative: L2 Pooling

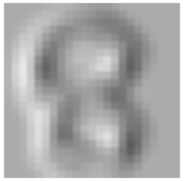
- L2 norm of a neighborhood of convolutional layer outputs
- Softer version of max pooling
  - Harder to differentiate



Local Contrast Normalization

- Standardize output of convolutional layer using mean & variability estimated from neighboring outputs

- Simple Example:

f: 

$$f_{ij} = \frac{f_{ij} - \mu_{ij}}{\sigma_{ij}}$$

$$\mu_{ij} = \text{mean}\left\{ f_{i'j'} \,\middle|\, (i',j') \text{ close to } (i,j) \right\}$$

$$\sigma_{ij}^2 = \text{mean}\left\{ \left(f_{i'j'} - \mu_{i'j'} \right)^2 \,\middle|\, (i',j') \text{ close to } (i,j) \right\}$$

Biologically Inspired!

- Other examples in references below:

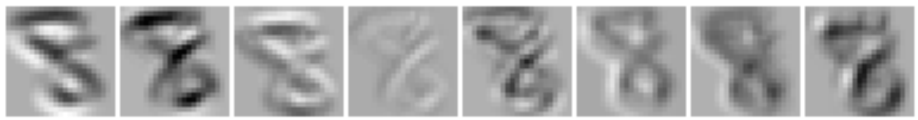
http://cs.nyu.edu/~fergus/tutorials/deep_learning_cvpr12/tutorial_p2_nnets_ranzato_short.pdf
http://www.cs.toronto.edu/~fritz/absps/imagenet.pdf

input (24x24x1)
max activation: 1, min: 0

Input

conv (23x23x8)
filter size 6x6x1, stride 1
max activation: 3.73813, min: -8.09174

Activations:



8 Convolutional Filters in 1st Layer

relu (23x23x8)
max activation: 3.73813, min: 0
max gradient: 0.00316, min: -0.00215

Activations:

Rectilinear Transform

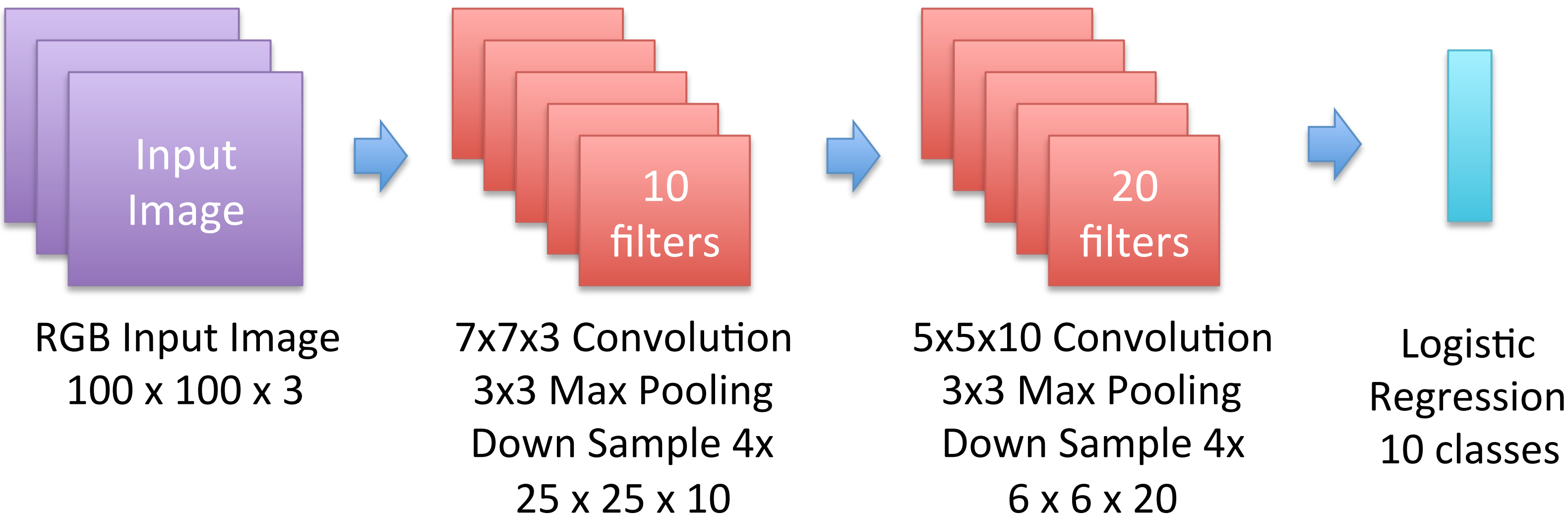
pool (11x11x8)
pooling size 2x2, stride 2
max activation: 3.29955, min: 0

Activations:

Max Pooling

http://cs.stanford.edu/people/karpathy/convnetjs/demo/mnist.html

Deep Convolutional Networks



- Stack multiple layers together
- Multiclass logistic regression at top
- Train using gradient descent

Down Sampling

- Adjacent Sliding Window Convolution
  - Yields output of same dimensions as input
- Good to compress into fewer pixels
  - Skip a few pixels for each convolution
- "Stride"
  - How far away next convolution is
  - No Down Sampling: Stride = 1
  - Down Sampling 2x: Stride = 2

Also Max Pooling

Online Demo

http://cs.stanford.edu/people/karpathy/convnetjs/demo/mnist.html

ImageNET

- Object recognition competition (2012)
  - 1.5 Million Labeled Training Examples
  - ≈1000 classes

Leopard

Mushroom

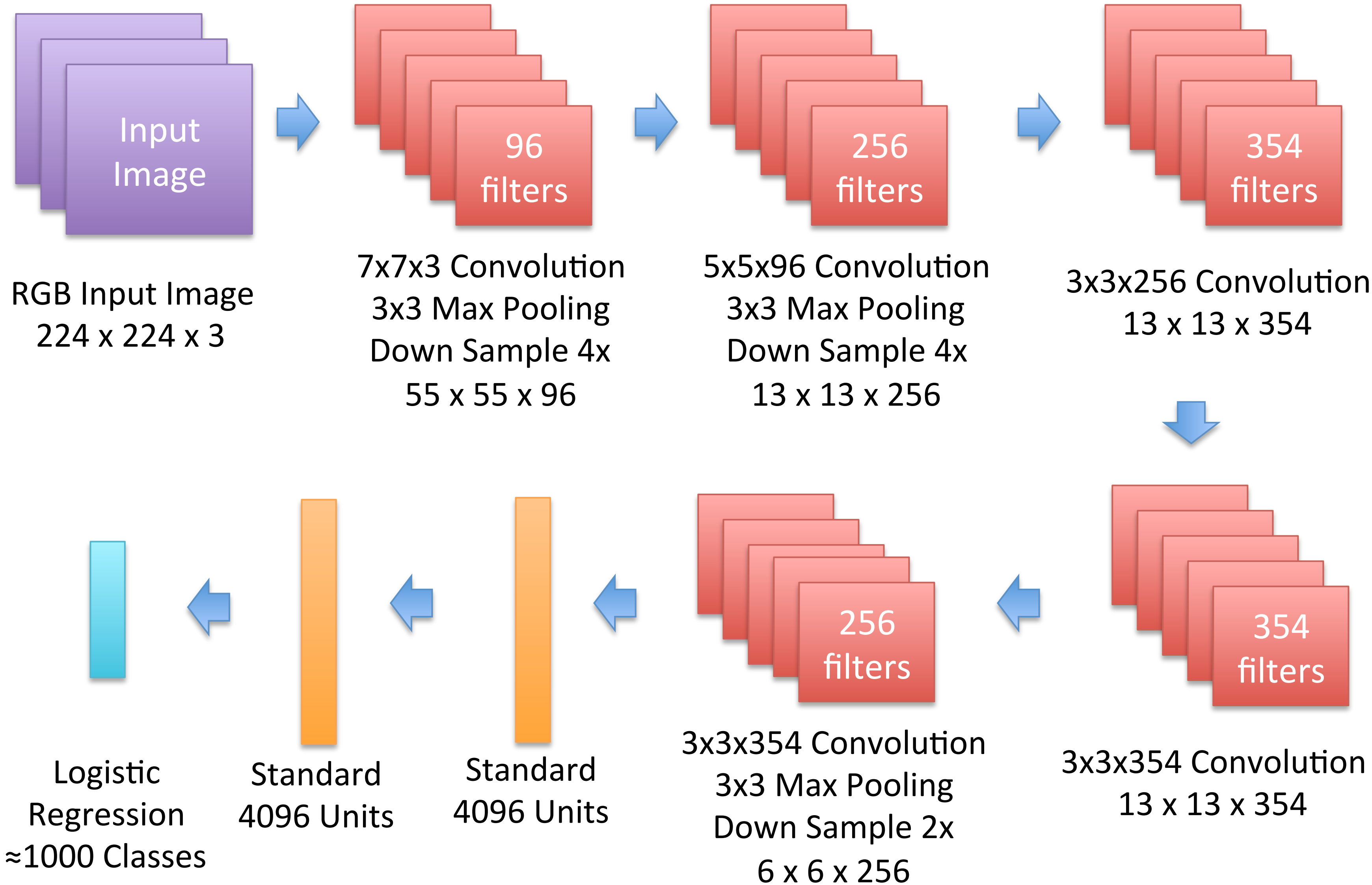
Mite

http://www.image-net.org/

Deep Convolutional Net for ImageNET

- 7 Hidden Layers
  - 5 Convolutional
  - 2 Regular
- Multiclass Logistic Regression at top
- Trained using stochastic gradient descent
  - And a lot of tricks
- Won the 2012 ImageNET competition

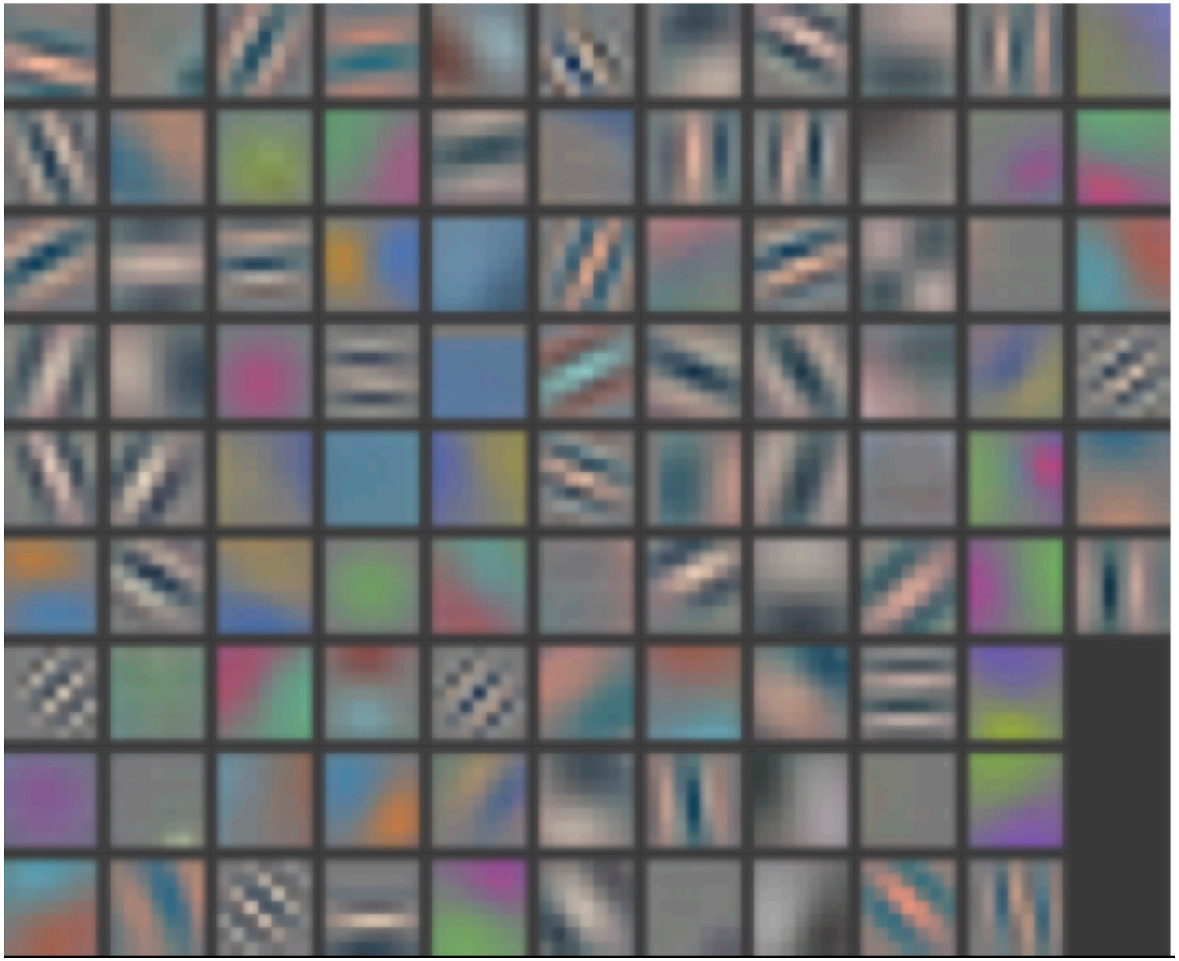
http://www.cs.toronto.edu/~fritz/absps/imagenet.pdf
http://www.image-net.org/challenges/LSVRC/2012/results.html



http://www.image-net.org/

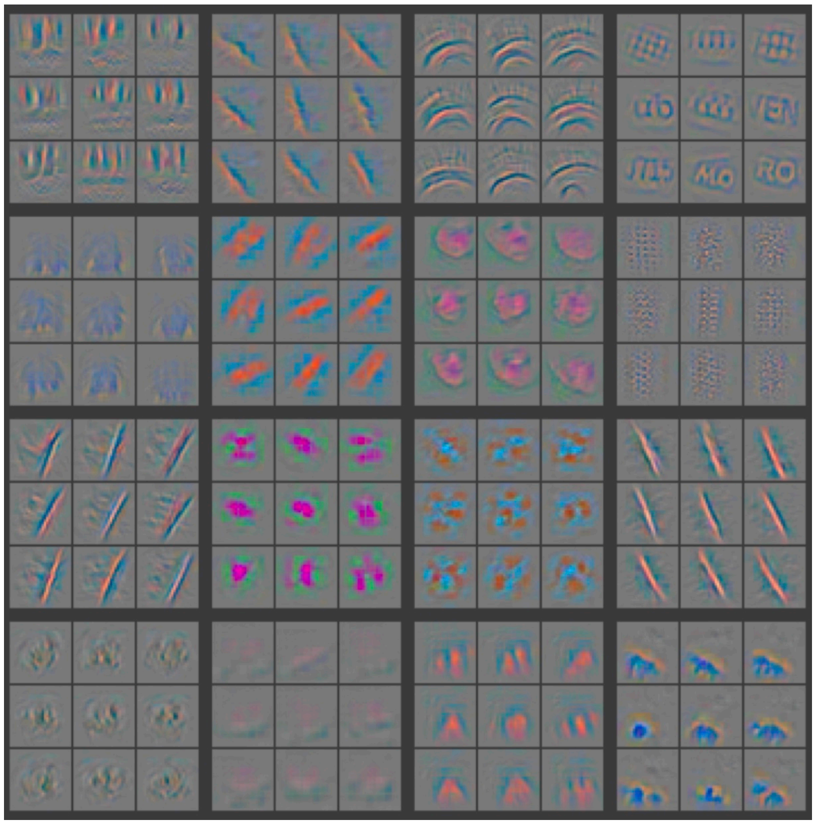
http://www.cs.toronto.edu/~fritz/absps/imagenet.pdf
http://ftp.cs.nyu.edu/~fergus/papers/zeilerECCV2014.pdf

Visualizing CNN (Layer 1)

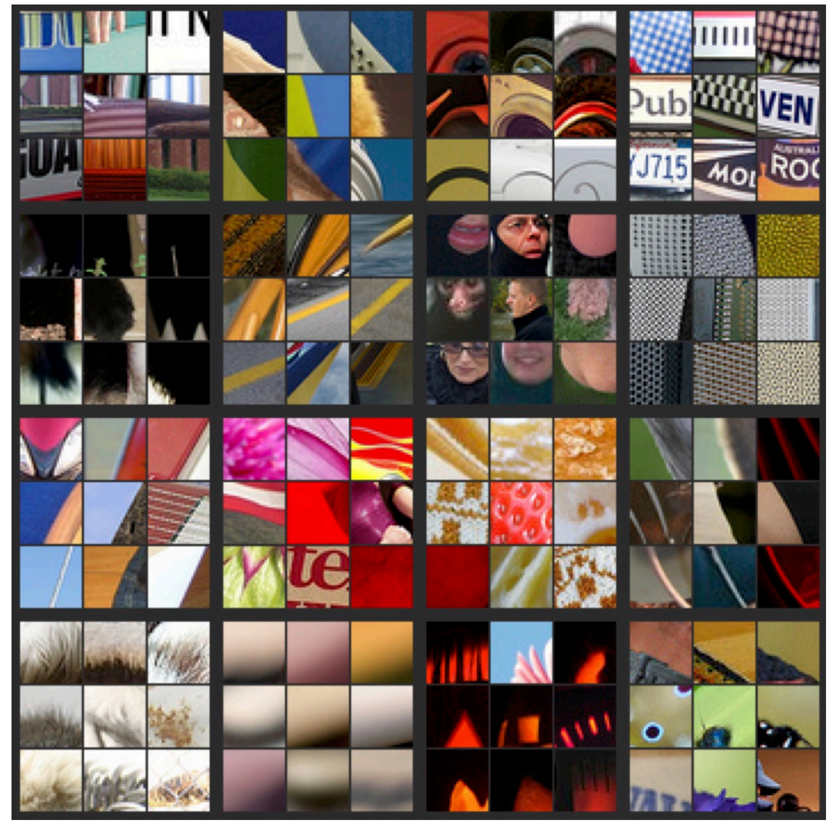


http://cs.nyu.edu/~fergus/papers/zeilerECCV2014.pdf
http://cs.nyu.edu/~fergus/presentations/nips2013_final.pdf

Visualizing CNN (Layer 2)



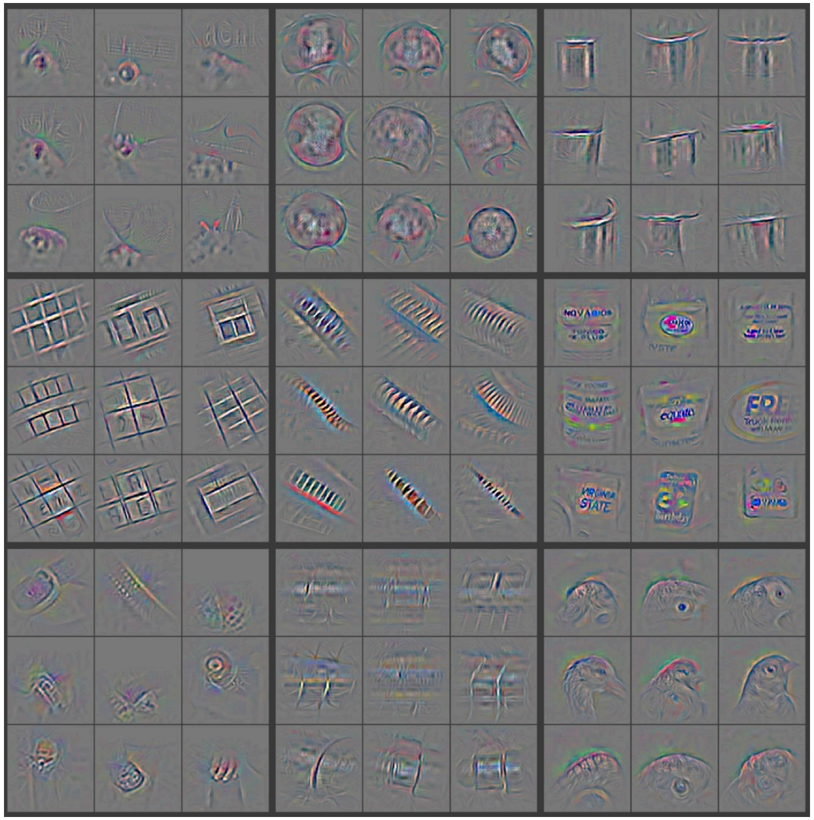
Part that Triggered Filter



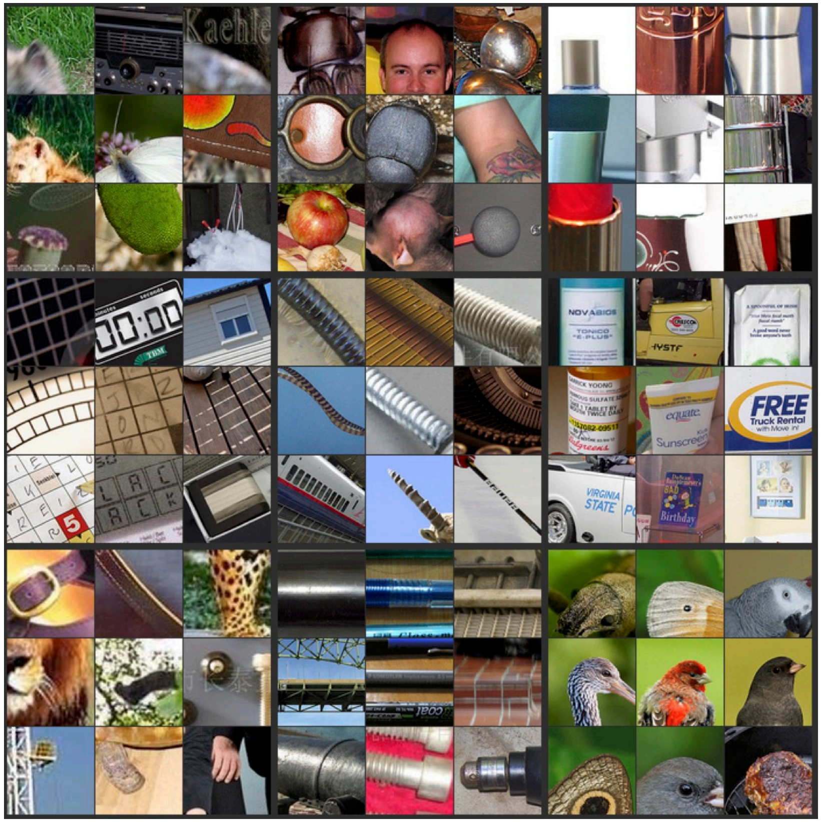
Top Image Patches

http://cs.nyu.edu/~fergus/papers/zeilerECCV2014.pdf
http://cs.nyu.edu/~fergus/presentations/nips2013_final.pdf

Visualizing CNN (Layer 3)



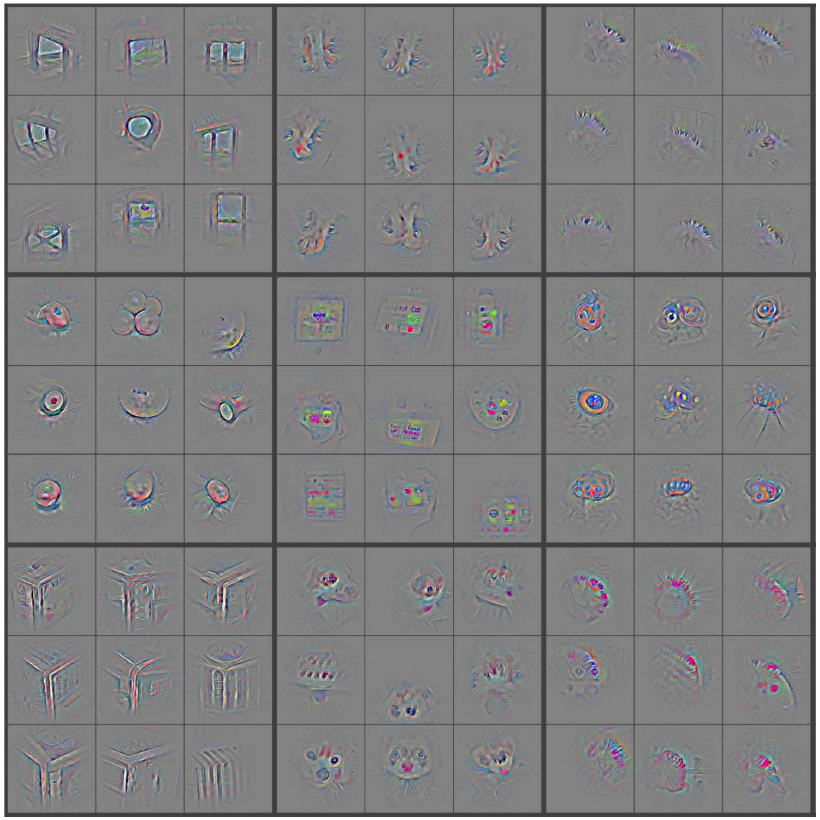
Part that Triggered Filter



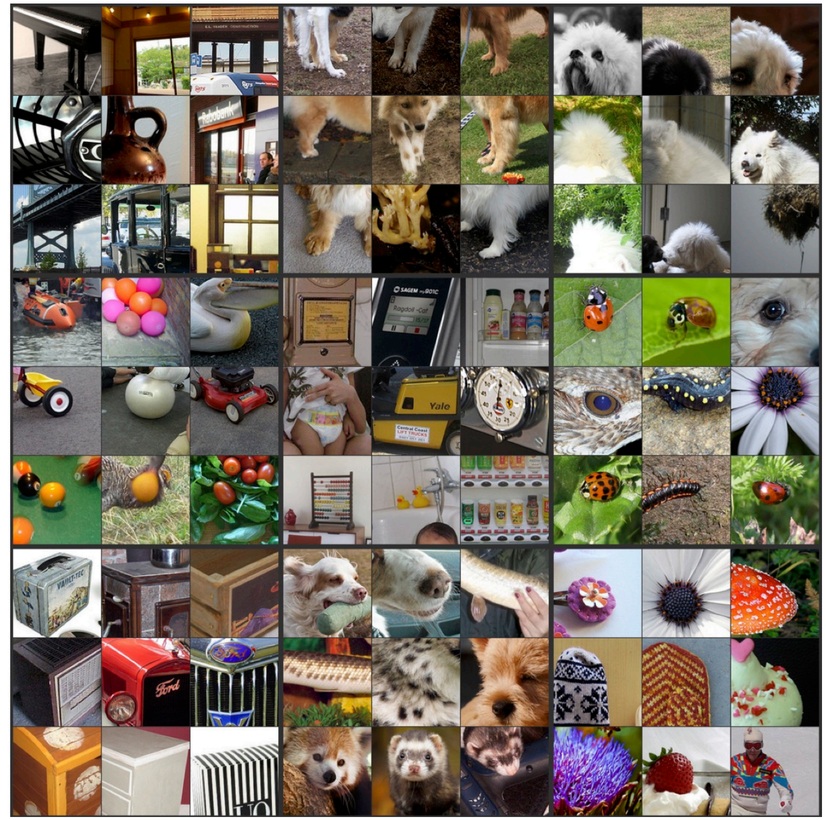
Top Image Patches

http://cs.nyu.edu/~fergus/papers/zeilerECCV2014.pdf
http://cs.nyu.edu/~fergus/presentations/nips2013_final.pdf

Visualizing CNN (Layer 4)



Part that Triggered Filter



Top Image Patches

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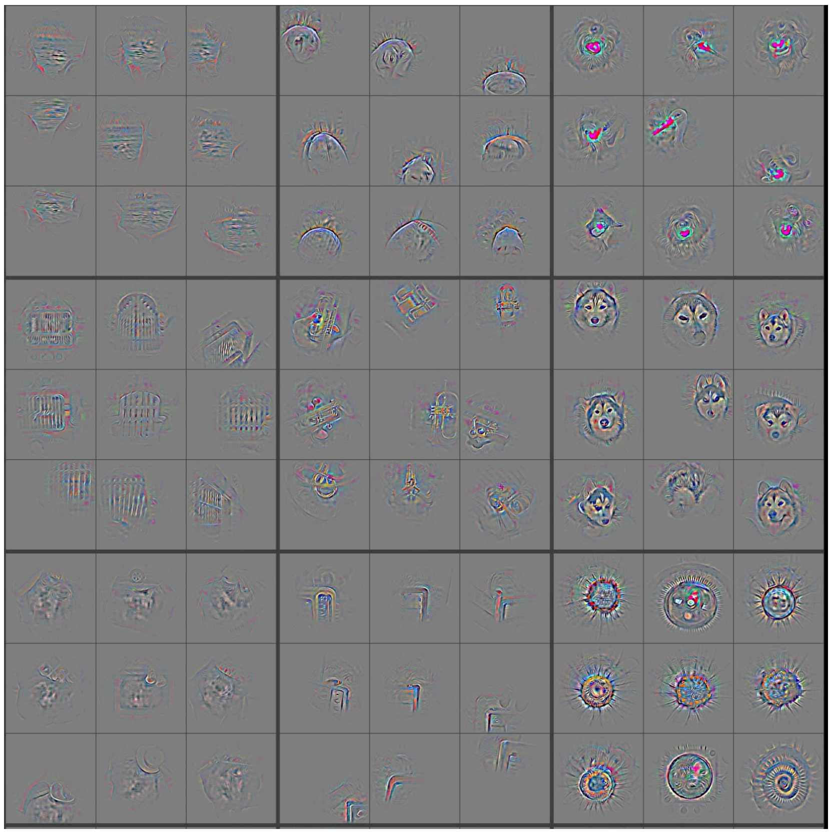
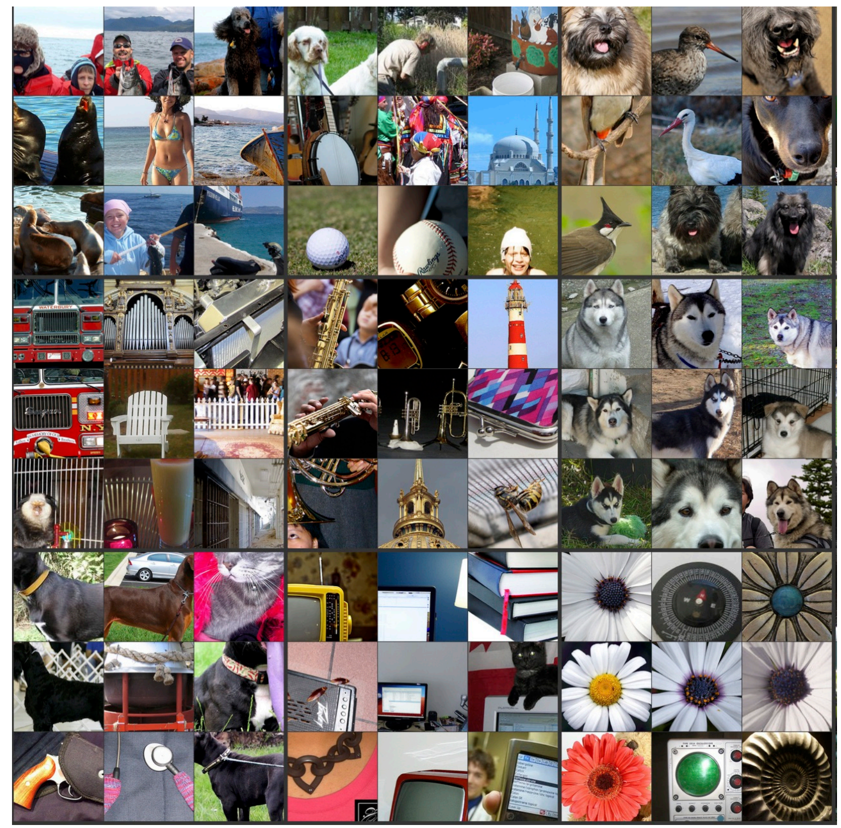
http://cs.nyu.edu/~fergus/presentations/nips2013_final.pdf

Visualizing CNN (Layer 5)



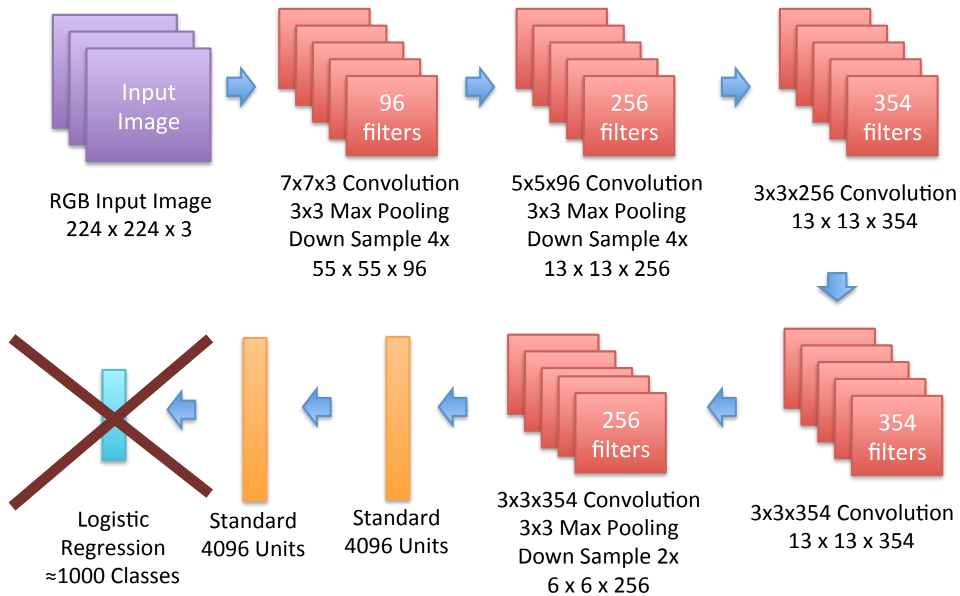
Part that Triggered Filter



Top Image Patches

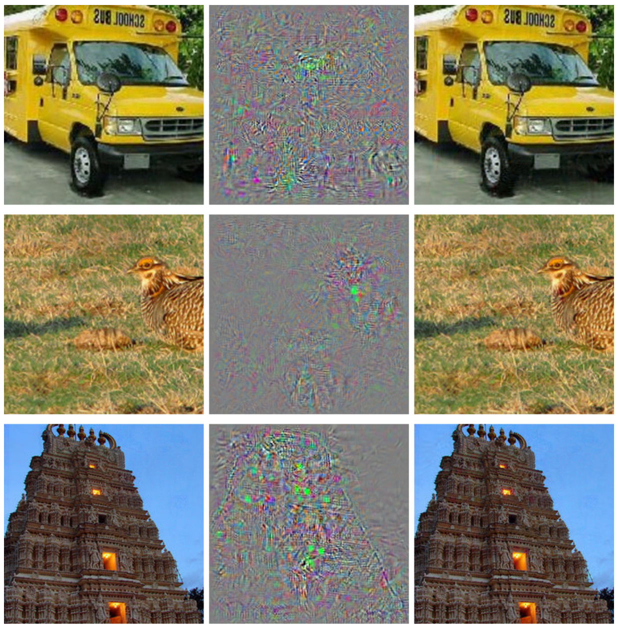
http://cs.nyu.edu/~fergus/papers/zeilerECCV2014.pdf
http://cs.nyu.edu/~fergus/presentations/nips2013_final.pdf

Use Hidden Layers as Features



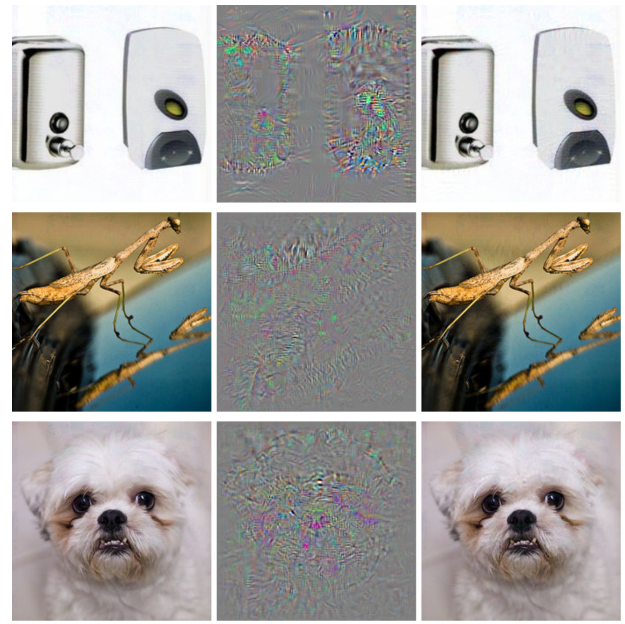
- Stack hidden layer activations as new feature representation
- Train an SVM =)
- Generalize to other datasets

Failure Cases



Predicts Correctly

Predicts Incorrectly



Predicts Correctly

Predicts Incorrectly

http://arxiv.org/pdf/1312.6199v4.pdf

Outline For Today

- Introduction to Deep Learning
  - Learning Features for Predictive Modeling
- Deep Convolutional Networks
  - Very popular in Computer Vision
- **Tips for Training Deep Networks**
- Brief Overview of other Deep Networks

Training Deep Networks

- Deep Networks are extremely non-convex
  - Hard to train well
- Deep Networks have extremely high capacity
  - Many parameters, easy to overfit
- Real success stories only in the last 10 years
  - A lot of (annotated) data
  - Increase in computational power
  - Better bag of tricks

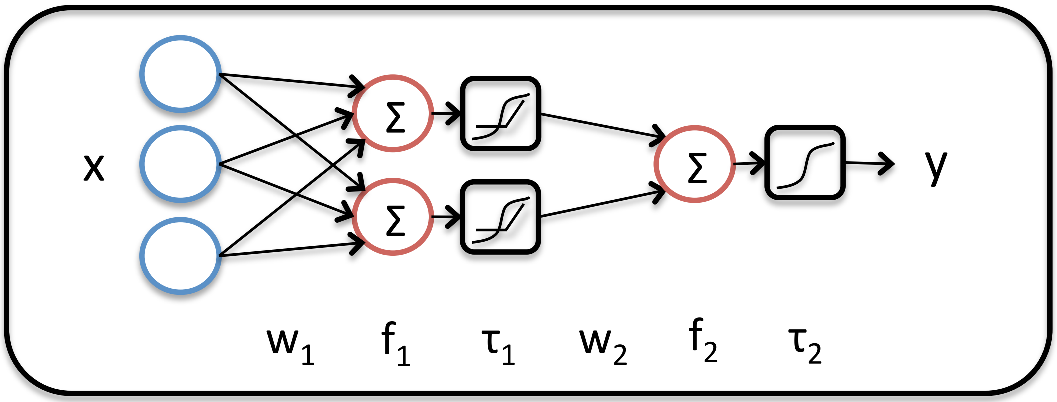
Stochastic Gradient Descent + Tricks!

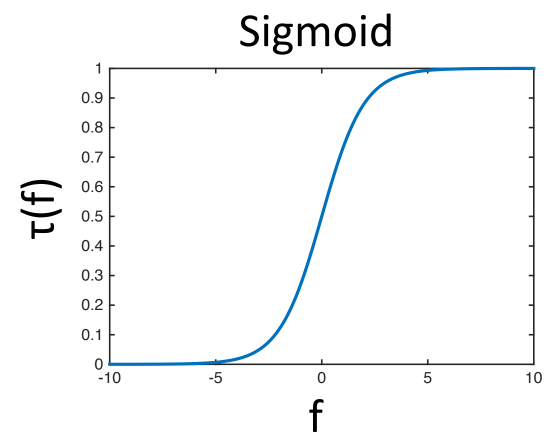
- Some related to choice of model/architecture
  - Rectilinear over sigmoid transfer functions
  - Local contrast normalization
  - Sparse Connections that enable parallelism
  - Ensemble of Deep Networks
- Rest are optimization techniques
  - Gradient Clamping
  - Mini-batching
  - Momentum
  - Adaptive Learning Rates
  - Random Initialization
  - Dropout

http://yyue.blogspot.com/2015/01/a-brief-overview-of-deep-learning.html

Rectilinear vs Sigmoid

(The Vanishing Gradient Problem)





$$\partial_{w_{1m}} L(y, \tau_2)$$

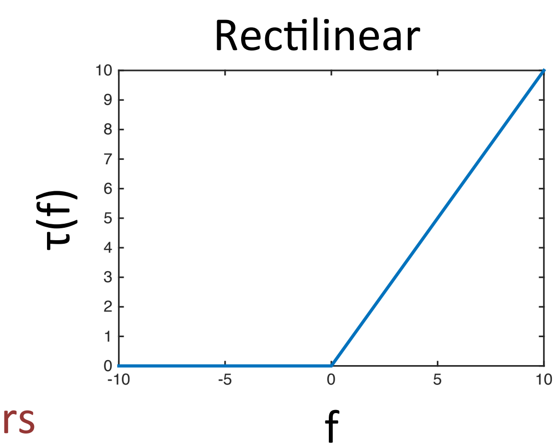
$$= \partial_{\tau_2} L(y, \tau_2) \partial_{f_2} \tau_2 \partial_{w_{1m}} f_2$$

$$= \partial_{\tau_2} L(y, \tau_2) \partial_{f_2} \tau_2 \partial_{\tau_{1m}} f_2 \underbrace{\partial_{f_{1m}} \tau_{1m}} \partial_{w_{1m}} f_{1m}$$

$$f_{1m} = w_{1m}^T x$$

Large f ➔ vanishing gradient

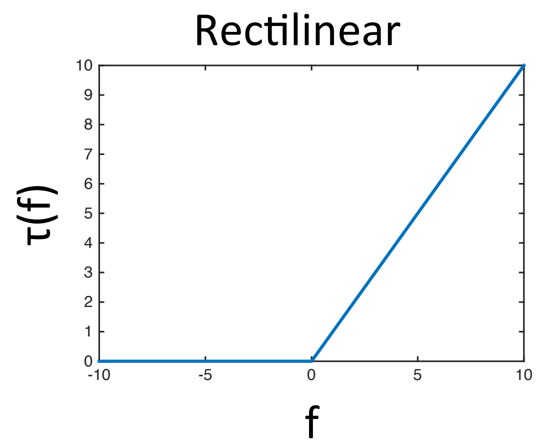
Compounded with more layers



Gradient Clamping

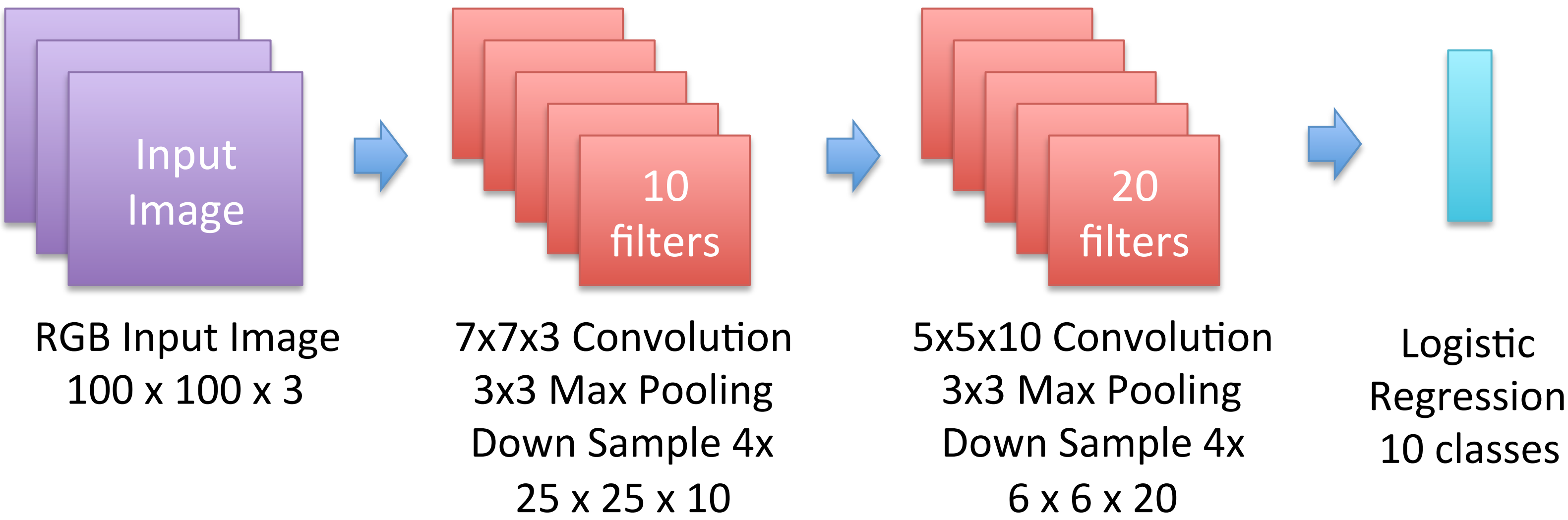
- Rectilinear functions can grow unbounded:
  - Gradients can get very large
  - Compounding effect in lower layers
    - Opposite of the vanishing gradient effect with sigmoids

- **Solution:** clamp gradients
  - E.g., clamp norm to 15



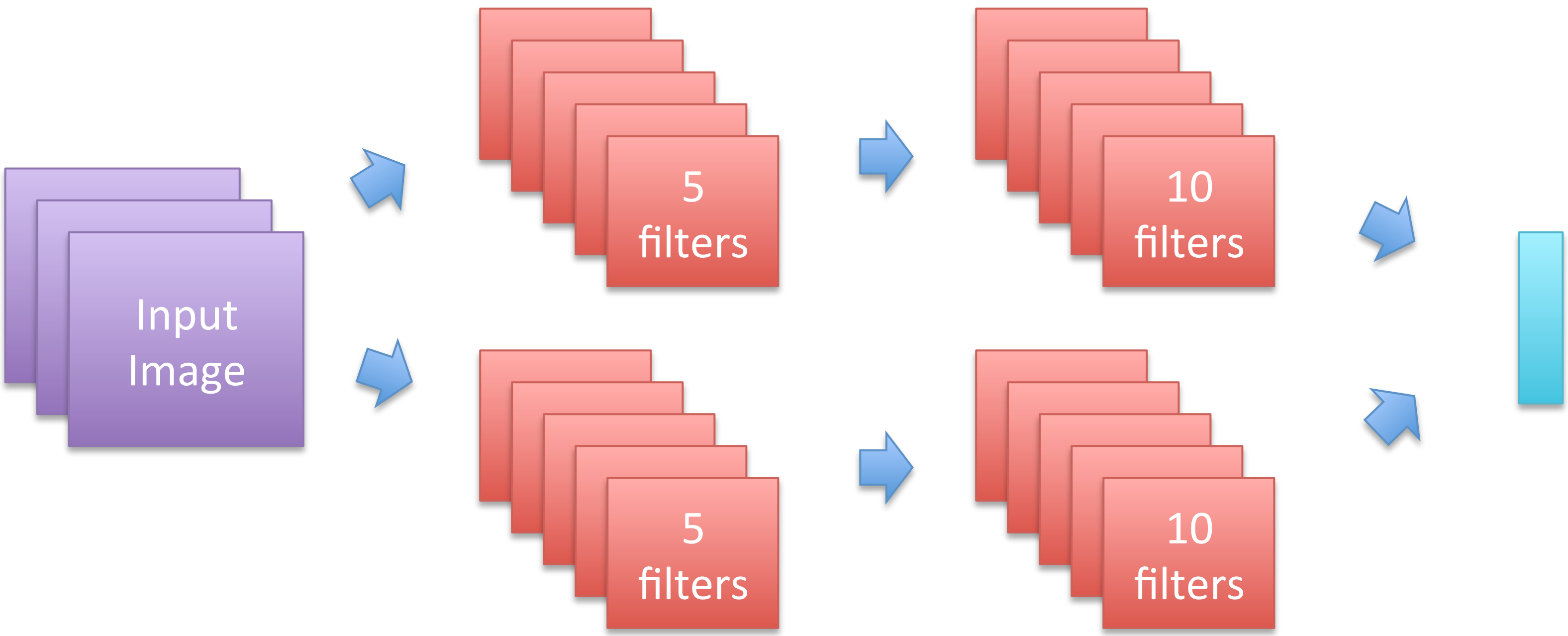
http://yyue.blogspot.com/2015/01/a-brief-overview-of-deep-learning.html

Dense Convolutional Networks



- Every Convolutional Layer uses every output from previous layer

Sparse Convolutional Networks



RGB Input Image
100 x 100 x 3

7x7x3 Convolution
3x3 Max Pooling
Down Sample 4x
25 x 25 x 10
Split Across 2 machines

5x5x5 Convolution
3x3 Max Pooling
Down Sample 4x
6 x 6 x 20
Split Across 2 Machines

Logistic
Regression
10 classes

Learning Rate & Momentum

$$w = w - \eta \partial_w$$ Gradient Descent

- If validation performance plateaus or gets worse
  - Divide learning rate by 2

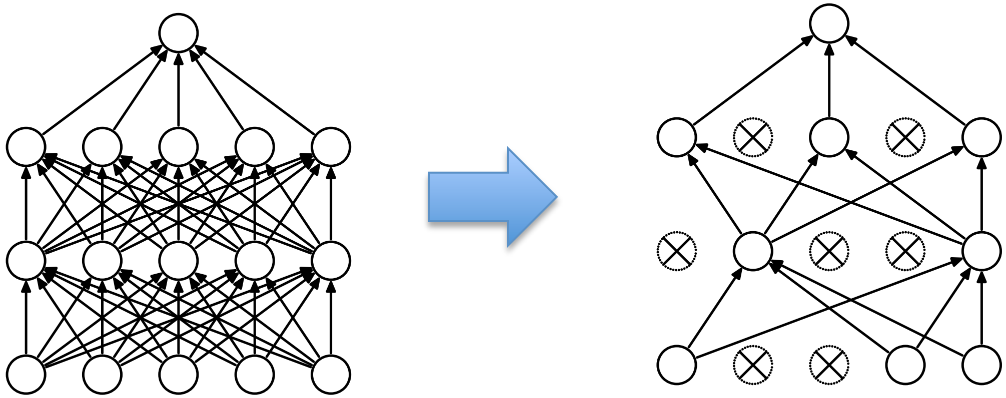
$$w = w - \eta \partial_w + \gamma m_w$$ Momentum

- Momentum is a weighted combination of recent gradient updates

http://www.cs.toronto.edu/~fritz/absps/momentum.pdf

Dropout

- Randomly turn off nodes during training



- Choose randomly for each SGD minibatch
  - Decorrelates node in each layer
  - Less overfitting

http://www.cs.toronto.edu/~rsalakhu/papers/srivastava14a.pdf

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Unsupervised Deep Learning

- Supervised = Learn Feature Encoding
  - Uses supervised label as signal
- Unsupervised = Also Learn Decoding
  - Uses reconstruction error as signal

RGB Input Image
100 x 100 x 3

7x7x3 Convolution
3x3 Max Pooling
Down Sample 4x
25 x 25 x 50

Combine Activations
to Produce Image
Minimize Squared Error
Between Output & Input

http://research.google.com/archive/unsupervised_icml2012.html

Unsupervised Deep Learning

- "Deep" dimensionality reduction
  - Can think of matrix factorization as "shallow" linear dimensionality reduction
- Encoding: convert image to features
  - Matrix Factorization: $z = Ux$
- Decoding: convert features to image
  - Matrix Factorization: $x = U^T z$

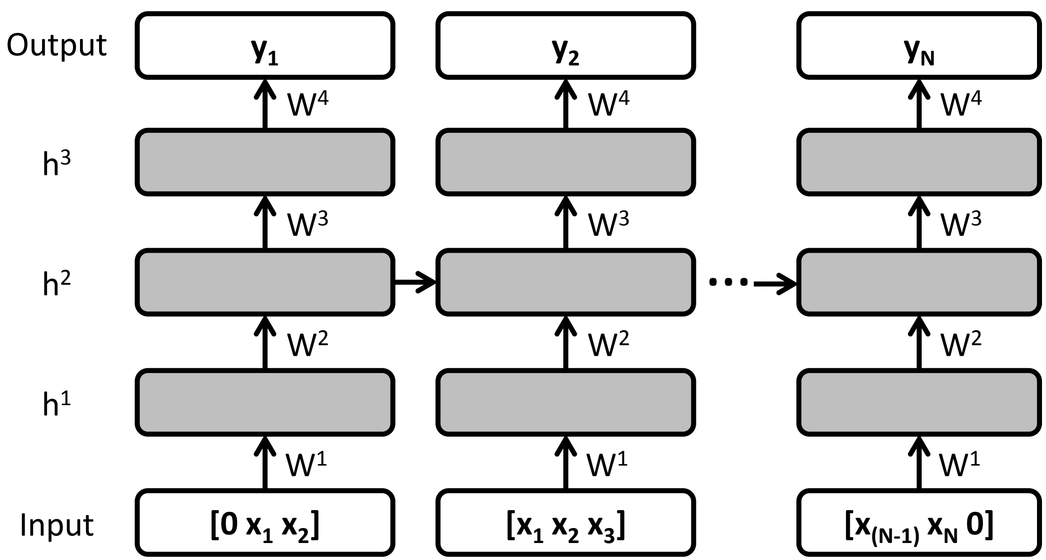
Deep Belief Networks

- Generative Model
- Encodes image as distribution over hidden state activations
- Can sample images given a setting of hidden states

http://www.cs.toronto.edu/~hinton/adi/

Deep Recurrent Networks

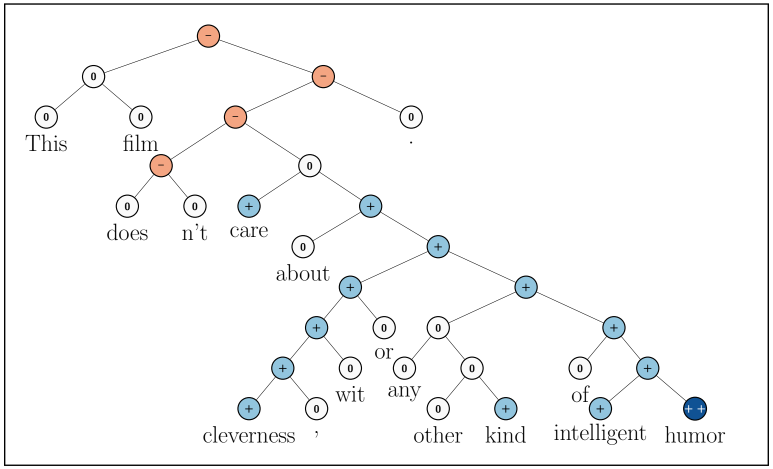
- Sequence Prediction
  - x & y are sequences



http://papers.nips.cc/paper/5346-sequence-to-sequence-learning-with-neural-networks.pdf
http://www1.icsi.berkeley.edu/~vinyals/Files/rnn_denoise_2012.pdf

Deep Recursive Networks

- Input: parse tree
- Output: sentiment



- Recursively instantiate model on parse tree
  - Each node takes the outputs of its children, and computes hidden layer activations as output
  - Logistic regression at the top

http://nlp.stanford.edu/sentiment/

Recap: Deep Learning

- Hierarchies (or layers) of non-linear transforms
  - Often interpreted as feature learning
  - Sometimes makes sense/visualizable
- Supervised training at the top layer
  - Unsupervised also possible (less successful)
- Train using stochastic gradient descent
  - But requires a lot of additional tricks
  - Also requires sufficient training data
- Nowhere close to general human cognition

Resources

- https://www.tensorflow.org/
- http://caffe.berkeleyvision.org/
- http://deeplearning.net/software/theano/
- http://torch.ch/
- https://code.google.com/p/cuda-convnet/
- http://cs.nyu.edu/~fergus/tutorials/deep_learning_cvpr12/
- http://deeplearning.net/tutorial/
- http://deeplearning.stanford.edu/tutorial/
- http://nlp.stanford.edu/sentiment/

Next Week

- Recent Applications
- Survey of Advanced Topics
- **Tonight:** Recitation on Advanced Optimization