

# DEEP LEARNING

## PART THREE - *DEEP GENERATIVE MODELS*

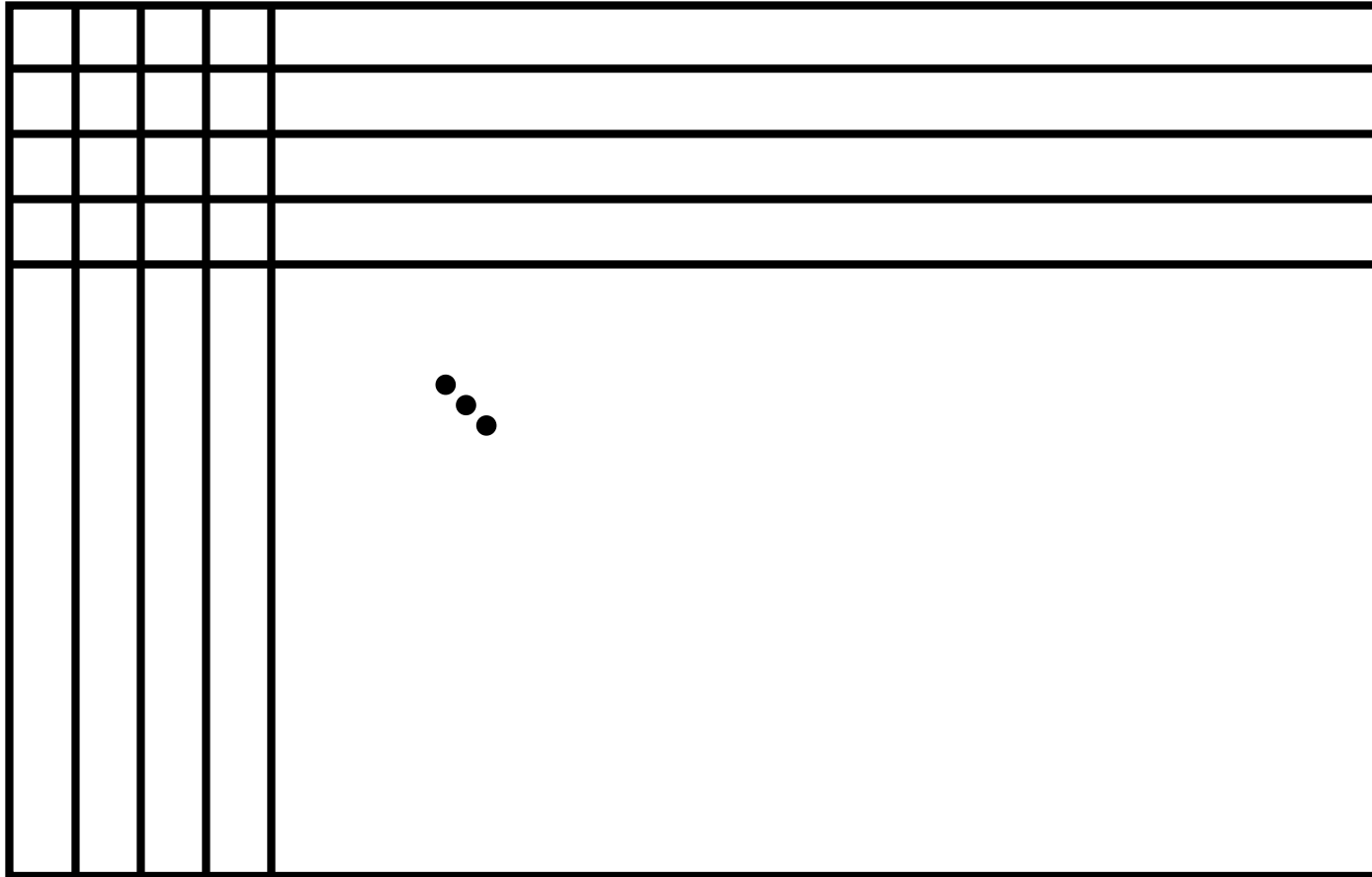


# GENERATIVE MODELS



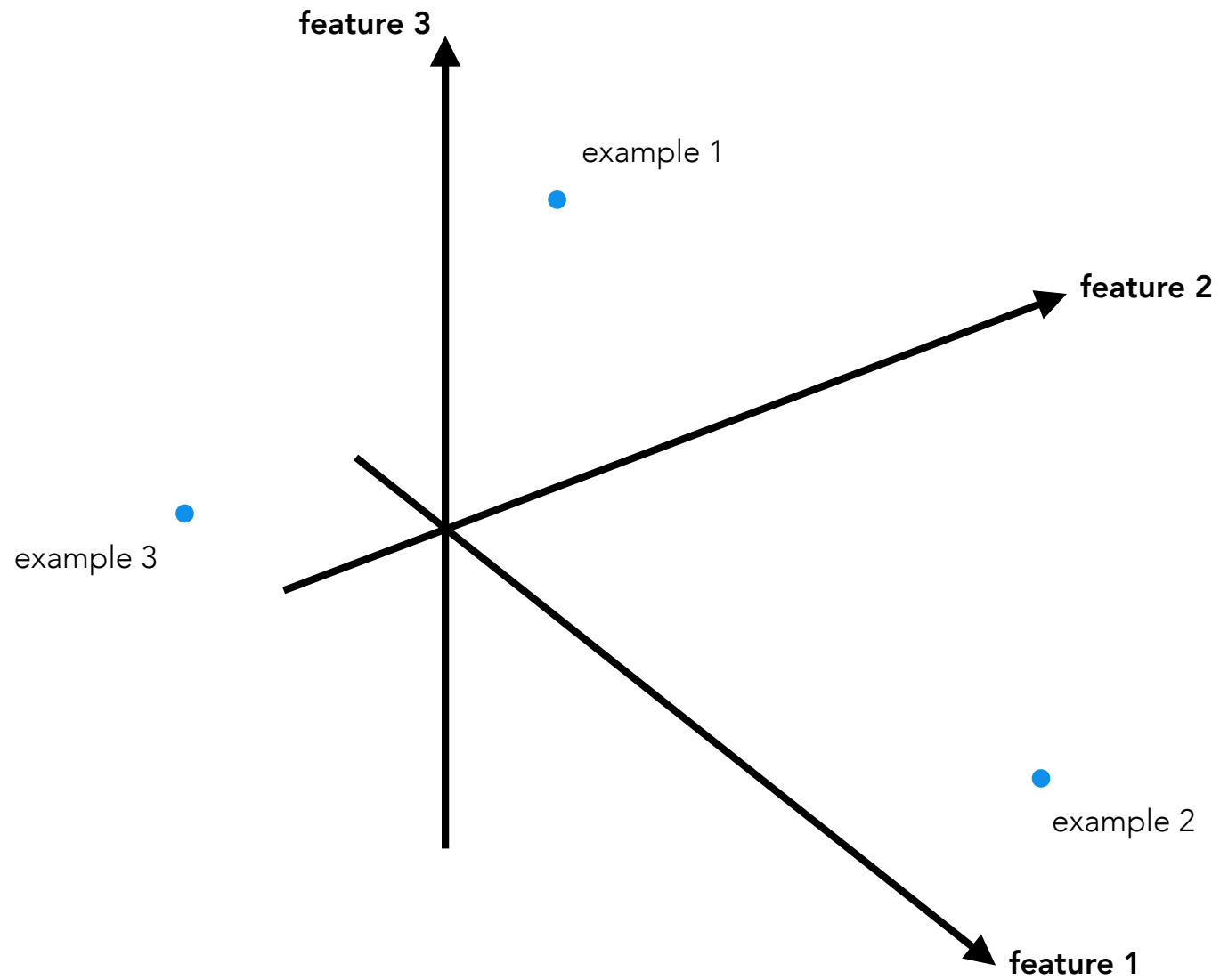
number of features

number of data examples



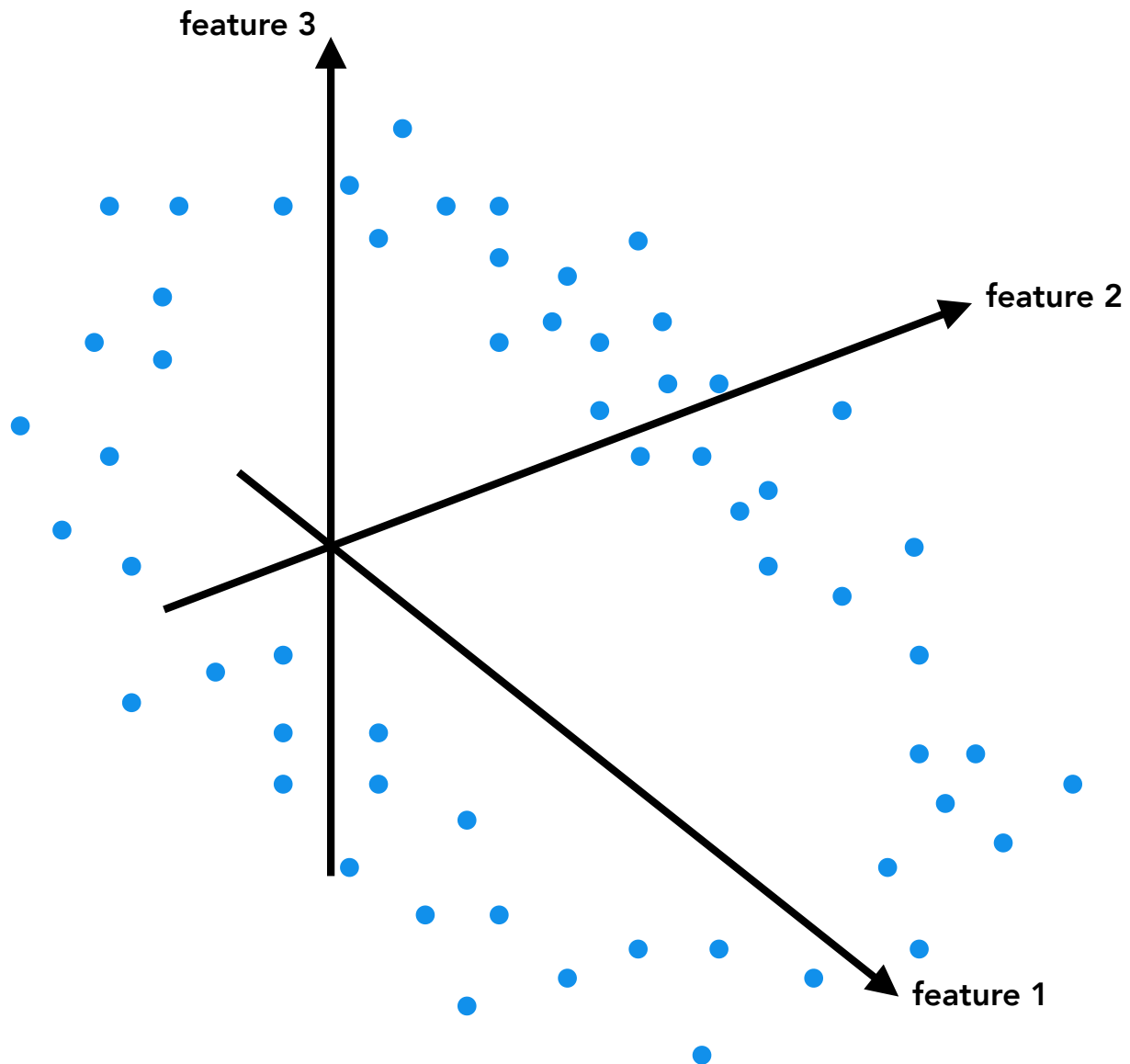
**DATA**





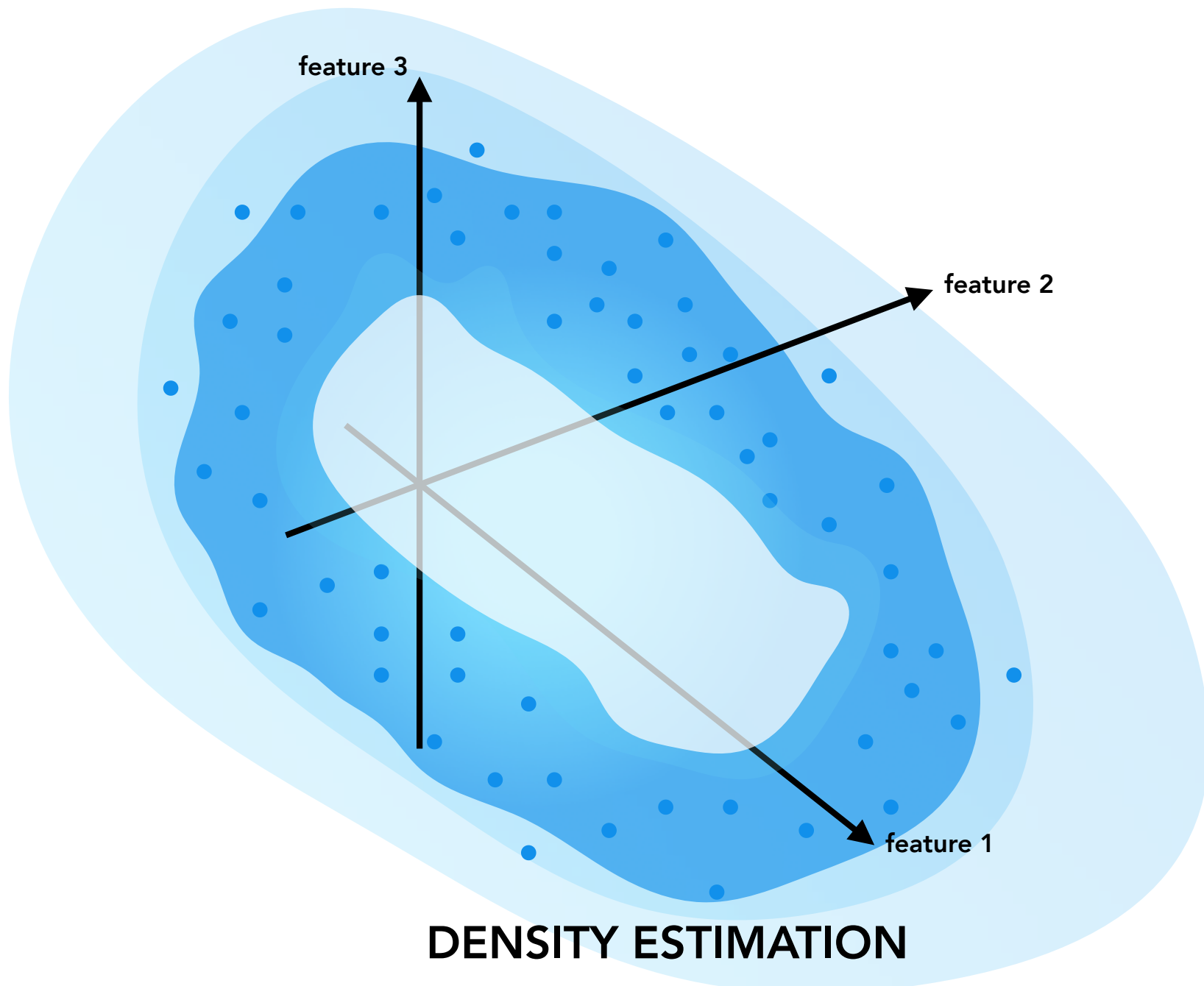
## DATA DISTRIBUTION





**EMPIRICAL DATA DISTRIBUTION**





## DENSITY ESTIMATION

*estimating the density of the empirical data distribution*



## **GENERATIVE MODEL**

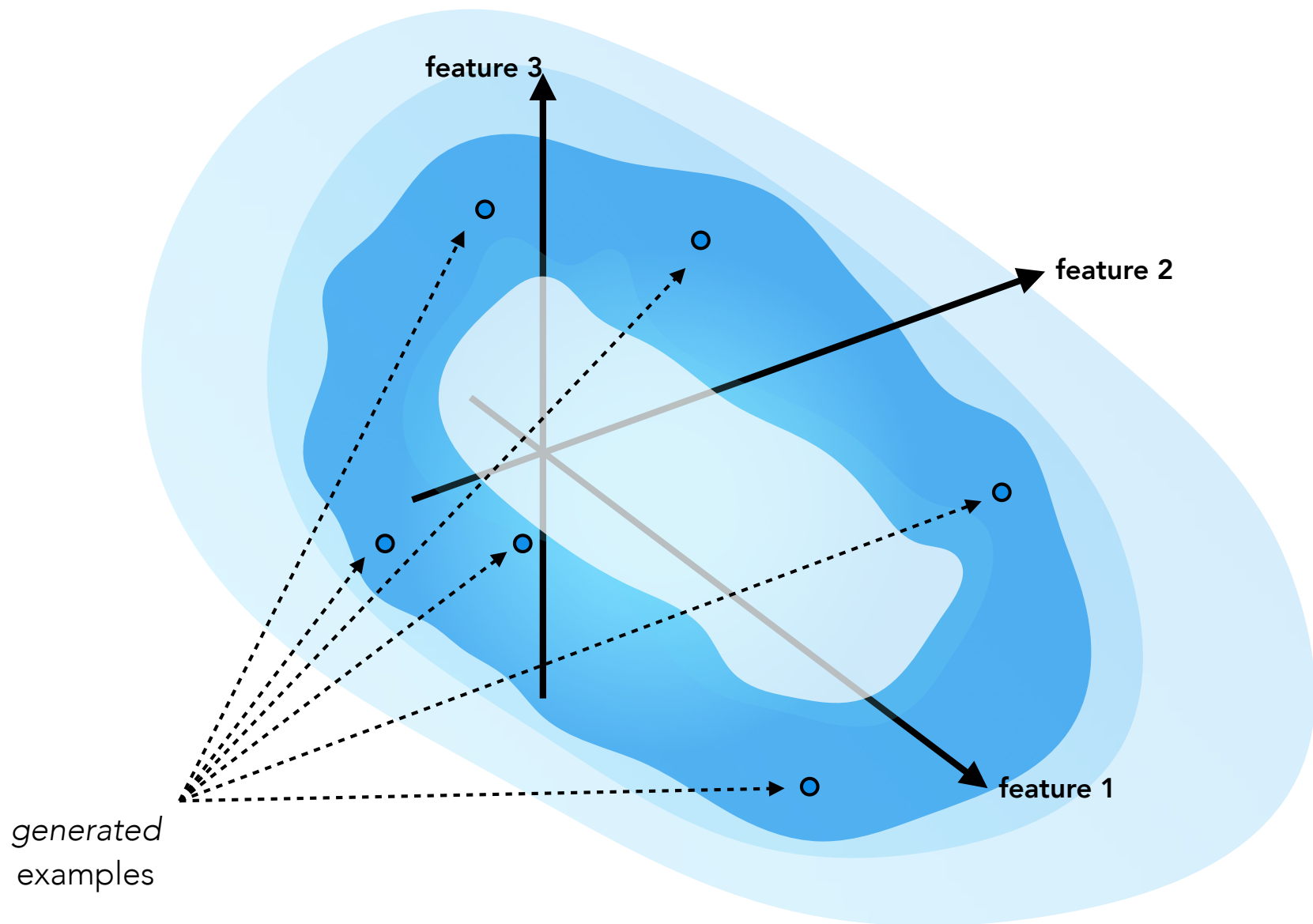
*a model of the density of the data distribution*



**why learn a generative model?**



generative models can **generate new data examples**



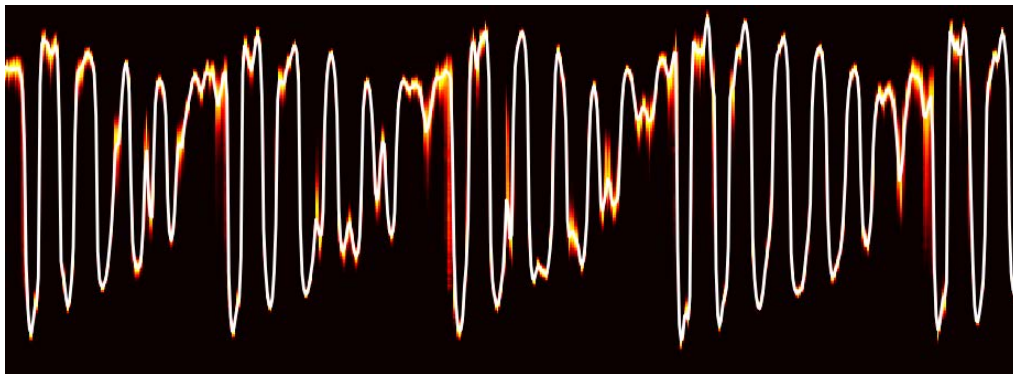




**Glow**, Kingma & Dhariwal, 2018



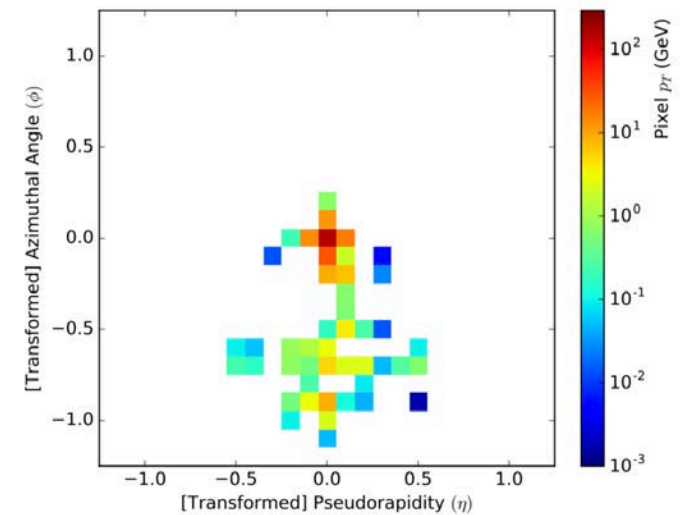
**BigGAN**, Brock et al., 2019



**WaveNet**, van den Oord et al., 2016



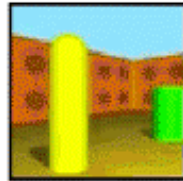
**MidiNet**, Yang et al., 2017



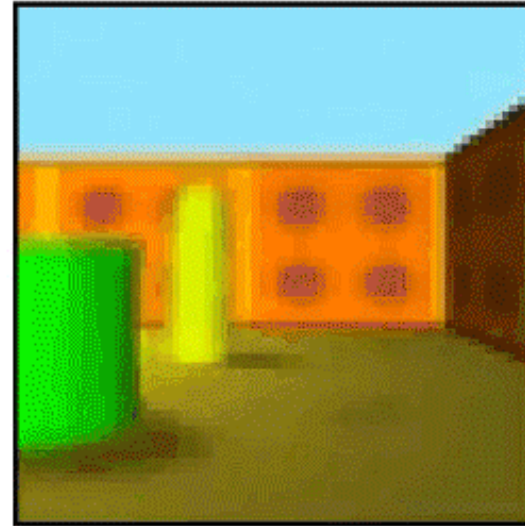
**Learning Particle Physics by Example**,  
de Oliveira et al., 2017



observation



neural rendering



**GQN**, Eslami *et al.*, 2018



Planning policies

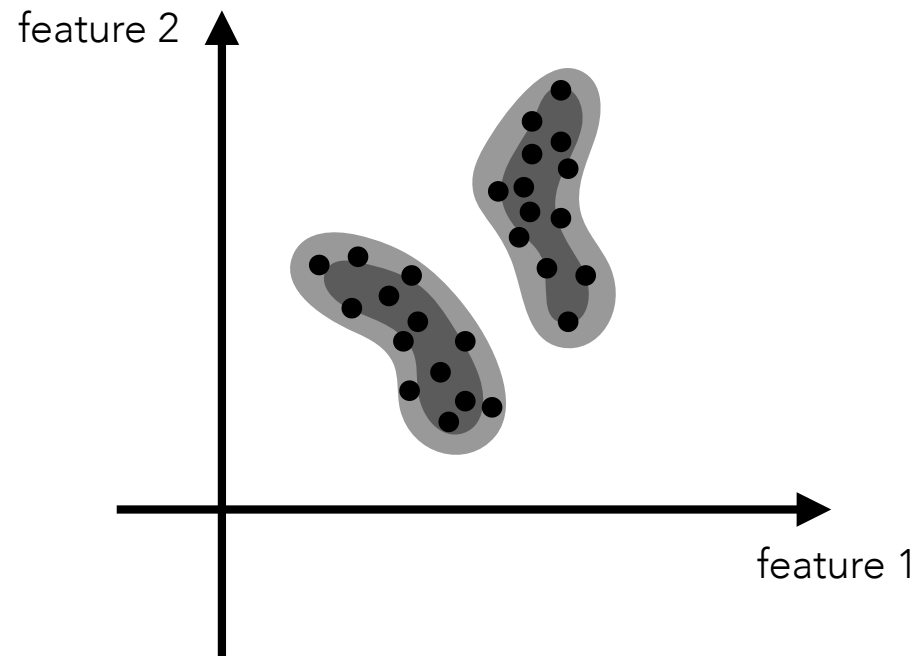


Long-term predictions

**PlaNet**, Hafner *et al.*, 2018

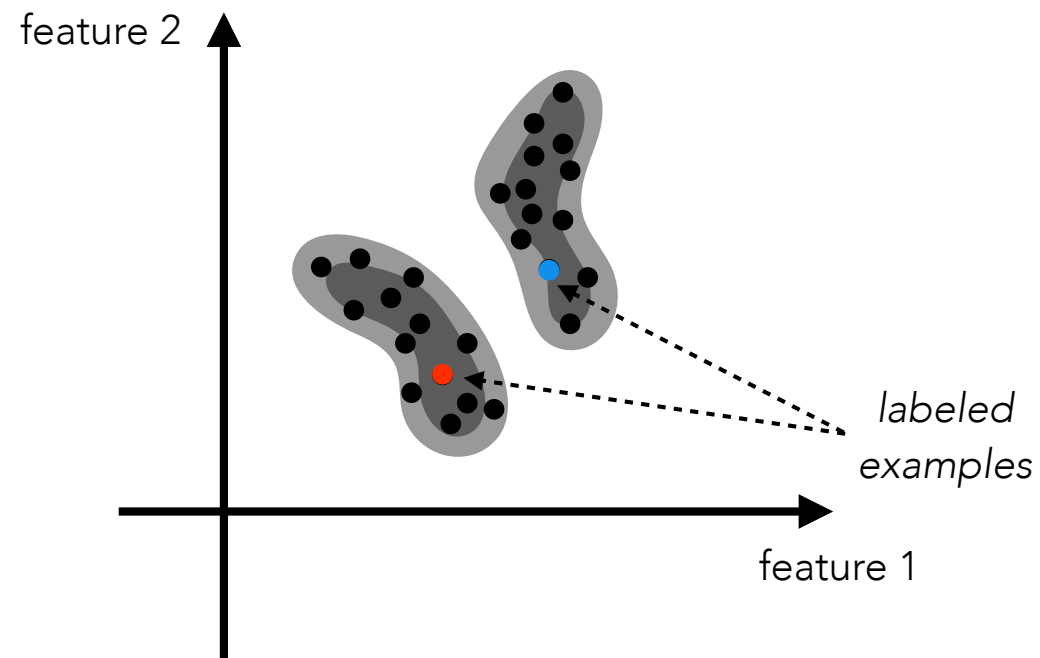


*generative models can **extract structure from data***



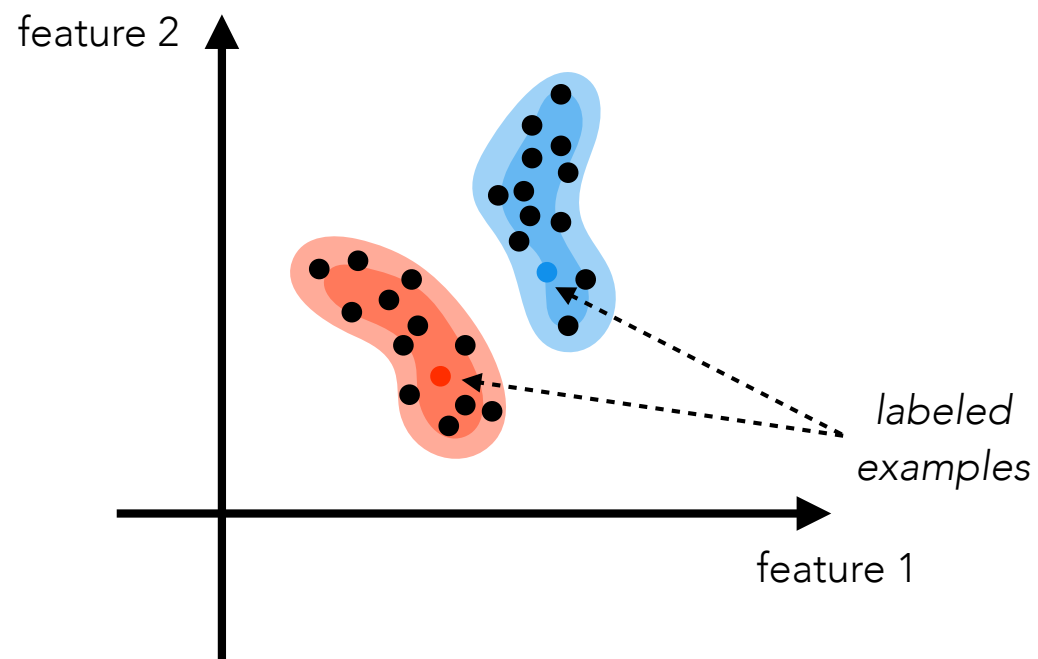


*generative models can **extract structure from data***





*generative models can **extract structure from data***



can make it easier to learn and generalize on new tasks





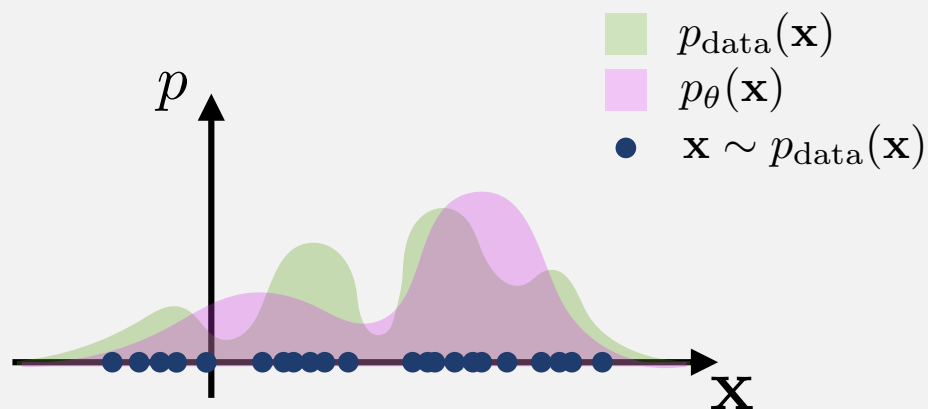


# modeling the data distribution

data:  $p_{\text{data}}(\mathbf{x})$

model:  $p_{\theta}(\mathbf{x})$

parameters:  $\theta$



## maximum likelihood estimation

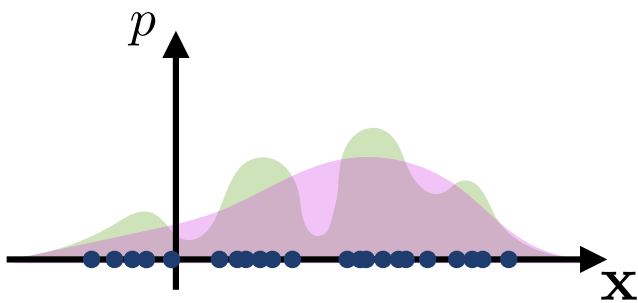
find the model that assigns the maximum likelihood to the data

$$\begin{aligned}\theta^* &= \arg \min_{\theta} D_{KL}(p_{\text{data}}(\mathbf{x}) || p_{\theta}(\mathbf{x})) \\ &= \arg \min_{\theta} \mathbb{E}_{p_{\text{data}}(\mathbf{x})} [\log p_{\text{data}}(\mathbf{x}) - \log p_{\theta}(\mathbf{x})] \\ &= \arg \max_{\theta} \mathbb{E}_{p_{\text{data}}(\mathbf{x})} [\log p_{\theta}(\mathbf{x})] \approx \frac{1}{N} \sum_{i=1}^N \log p_{\theta}(\mathbf{x}^{(i)})\end{aligned}$$

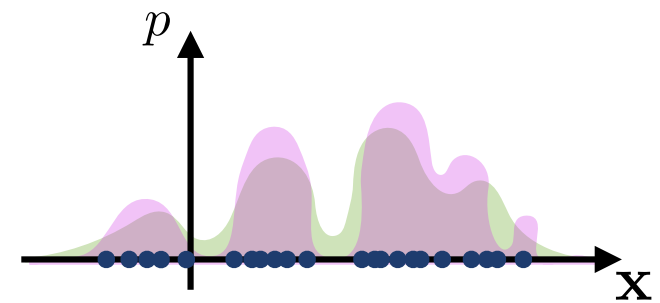
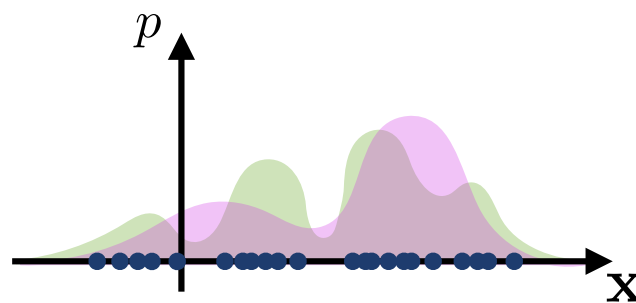


# bias-variance trade-off

- $p_{\text{data}}(\mathbf{x})$
- $p_{\theta}(\mathbf{x})$
- $\mathbf{x} \sim p_{\text{data}}(\mathbf{x})$



large bias



large variance



model complexity



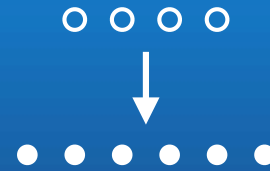
## ***deep generative model***

a generative model that uses deep neural networks  
to model the data distribution

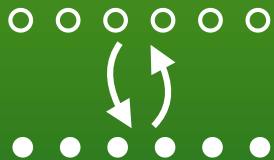




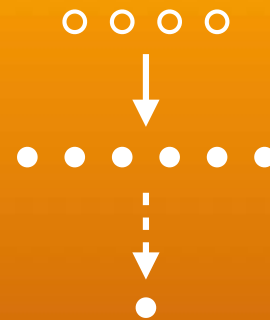
*autoregressive  
models*



*explicit  
latent variable models*

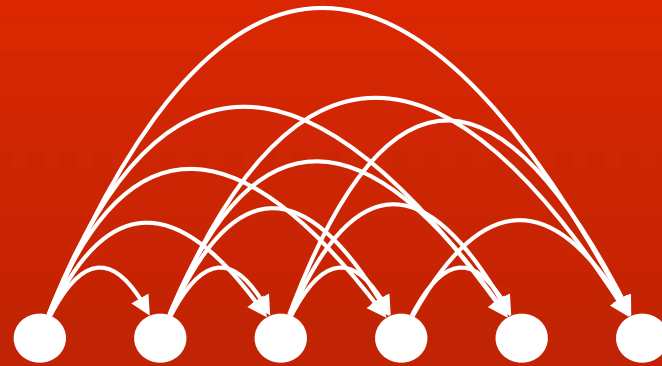


*invertible explicit  
latent variable models*



*implicit  
latent variable models*





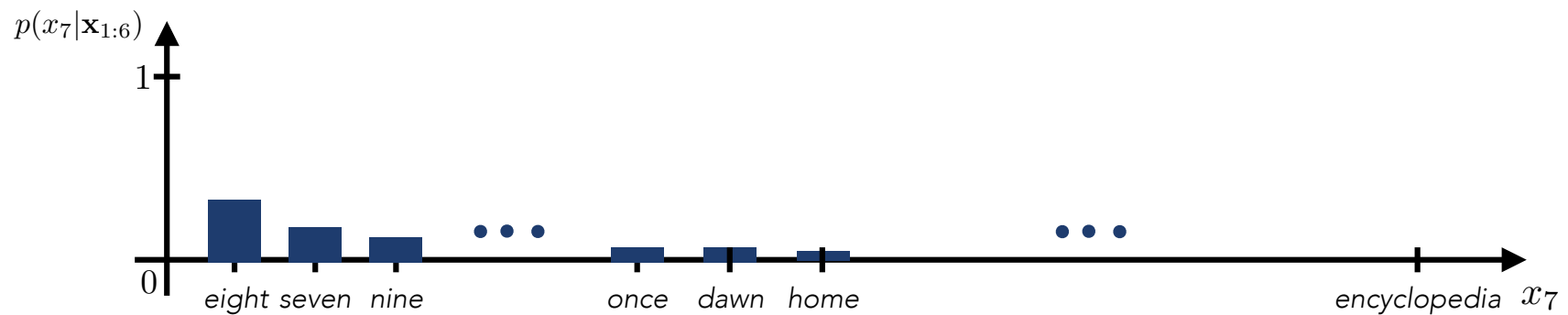
***autoregressive  
models***



## conditional probability distributions

<i>This</i>	<i>morning</i>	<i>I</i>	<i>woke</i>	<i>up</i>	<i>at</i>	
$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$

What is  $p(x_7|\mathbf{x}_{1:6})$ ?





*a data example*



$$p(\mathbf{x}) = p(x_1, x_2, \dots, x_M)$$



## chain rule of probability

*split the joint distribution into a product of conditional distributions*

$$\begin{array}{ccccccc} x_1 & x_2 & x_3 & & \dots & & x_M \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ p(\mathbf{x}) = p(x_1, x_2, \dots, x_M) \end{array}$$

$$p(a|b) = \frac{p(a, b)}{p(b)} \longrightarrow p(a, b) = p(a|b)p(b) \quad \text{definition of conditional probability}$$

recursively apply to  $p(x_1, x_2, \dots, x_M)$ :

$$\begin{aligned} p(x_1, x_2, \dots, x_M) &= p(x_1)p(x_2, \dots, x_M|x_1) \\ &\vdots \\ &= p(x_1)p(x_2|x_1) \dots p(x_M|x_1, \dots, x_{M-1}) \end{aligned}$$

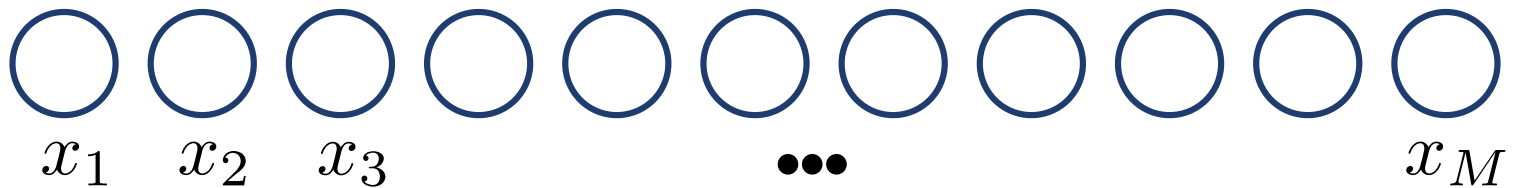
$$p(x_1, \dots, x_M) = \prod_{j=1}^M p(x_j|x_1, \dots, x_{j-1})$$

*note: conditioning order is arbitrary*



model the conditional distributions of the data

learn to **auto-regress** each value

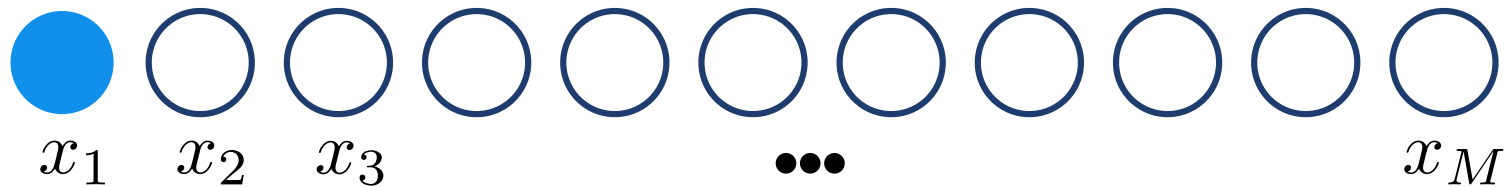




model the conditional distributions of the data

learn to **auto-regress** each value

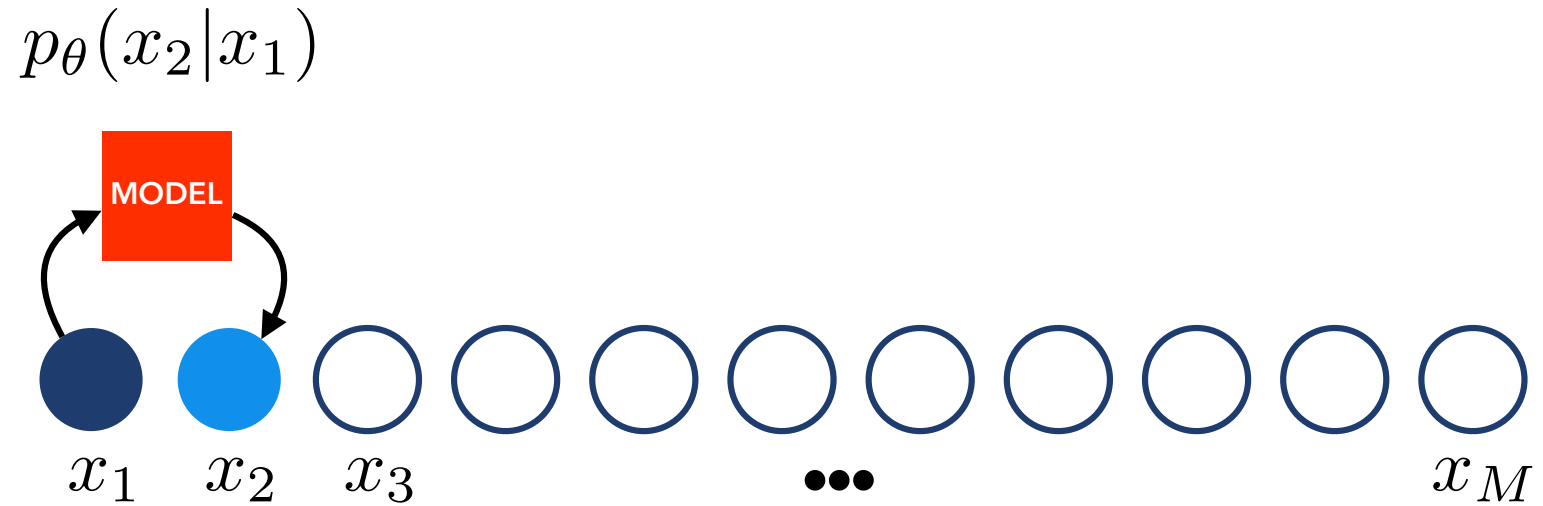
$$p_{\theta}(x_1)$$





model the conditional distributions of the data

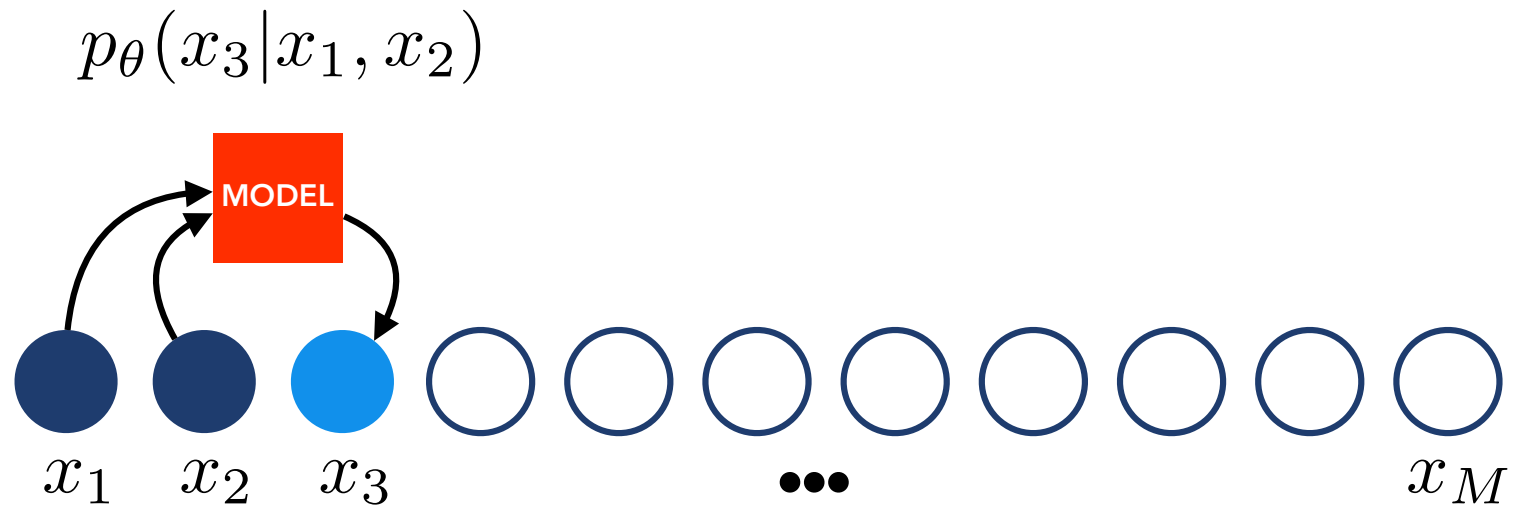
learn to **auto-regress** each value





model the conditional distributions of the data

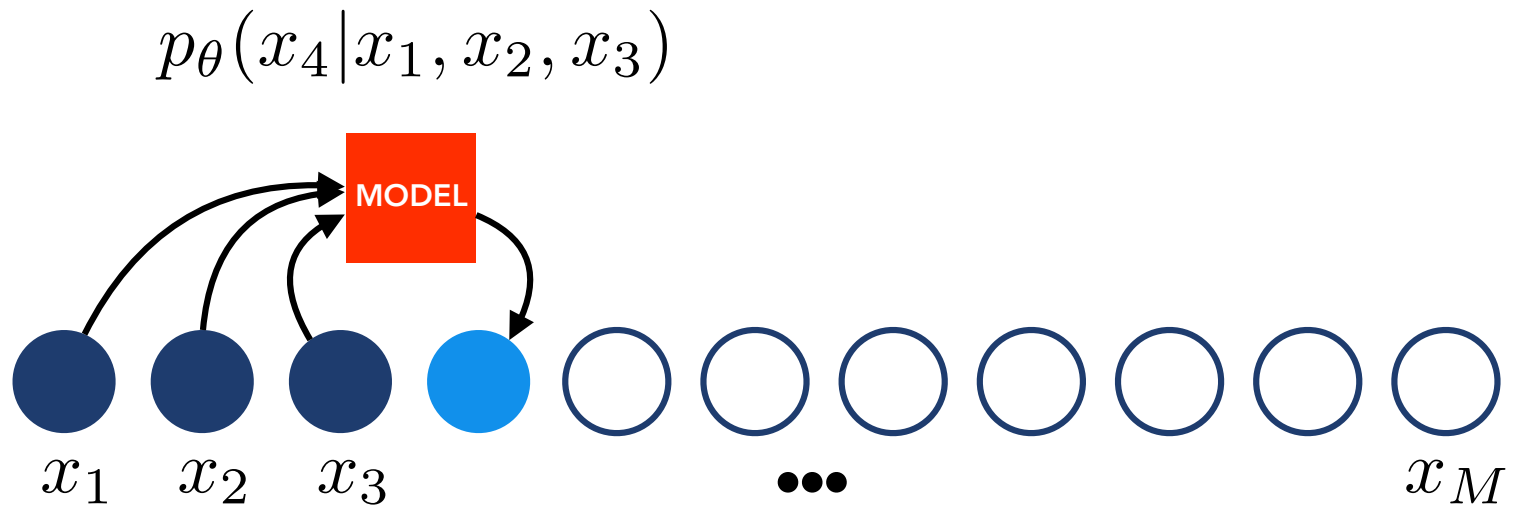
learn to **auto-regress** each value





model the conditional distributions of the data

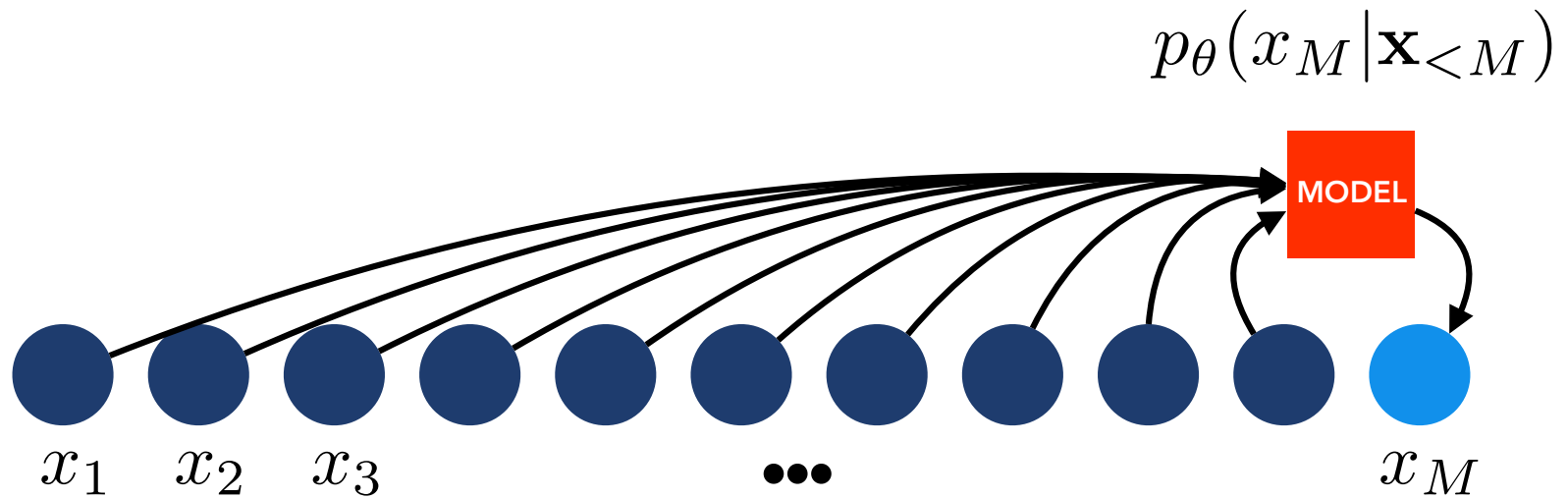
learn to **auto-regress** each value





model the conditional distributions of the data

learn to **auto-regress** each value





# maximum likelihood estimation

*maximize the log-likelihood (under the model) of the true data examples*

$$\theta^* = \arg \max_{\theta} \mathbb{E}_{p_{\text{data}}(\mathbf{x})} [\log p_{\theta}(\mathbf{x})] \approx \frac{1}{N} \sum_{i=1}^N \log p_{\theta}(\mathbf{x}^{(i)})$$

for auto-regressive models:

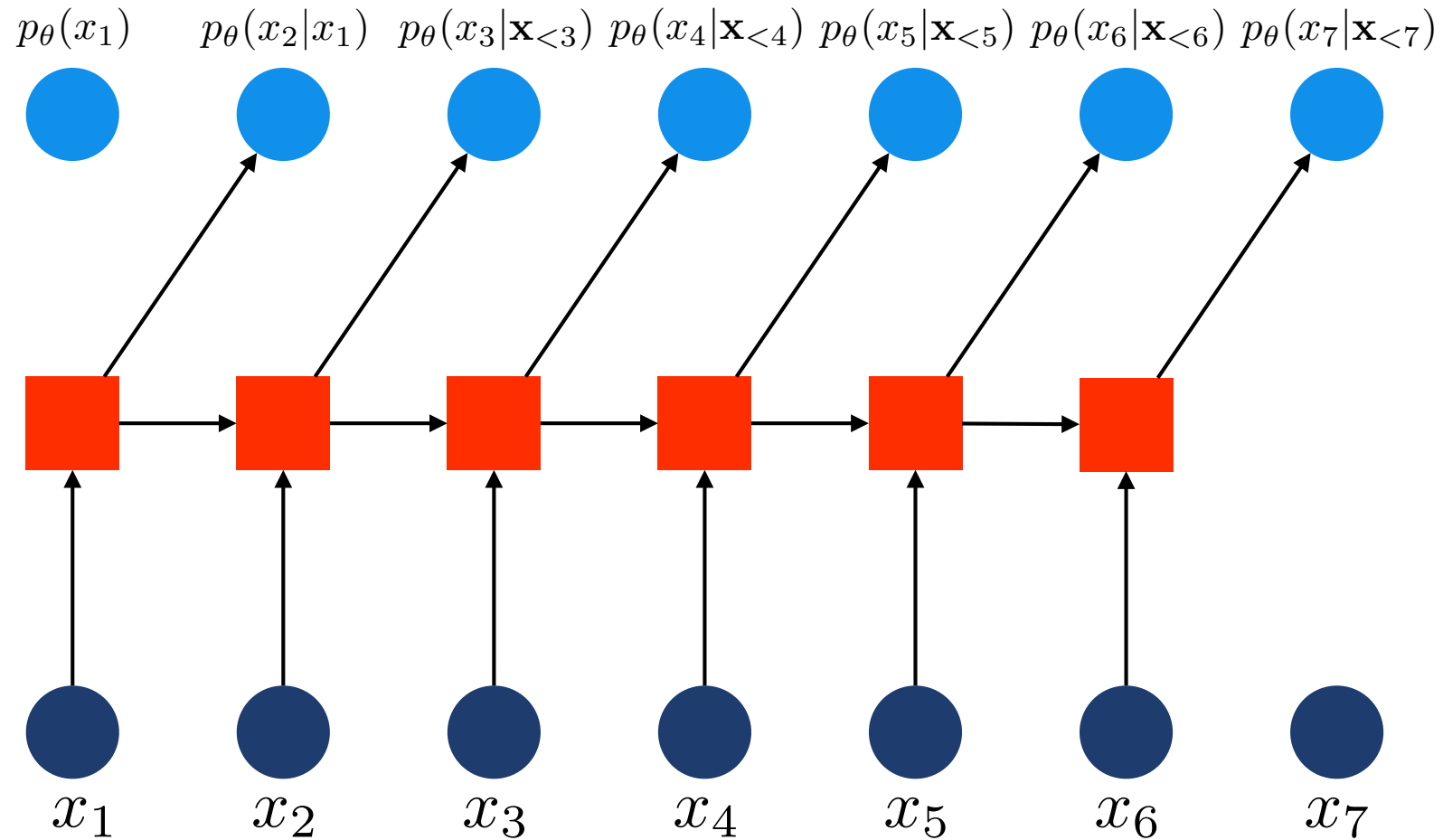
$$\begin{aligned} \log p_{\theta}(\mathbf{x}) &= \log \left( \prod_{j=1}^M p_{\theta}(x_j | \mathbf{x}_{<j}) \right) \\ &= \sum_{j=1}^M \log p_{\theta}(x_j | \mathbf{x}_{<j}) \end{aligned}$$

$$\theta^* = \arg \max_{\theta} \frac{1}{N} \sum_{i=1}^N \sum_{j=1}^M \log p_{\theta}(x_j^{(i)} | \mathbf{x}_{<j}^{(i)})$$



# models

can parameterize conditional distributions using a **recurrent neural network**

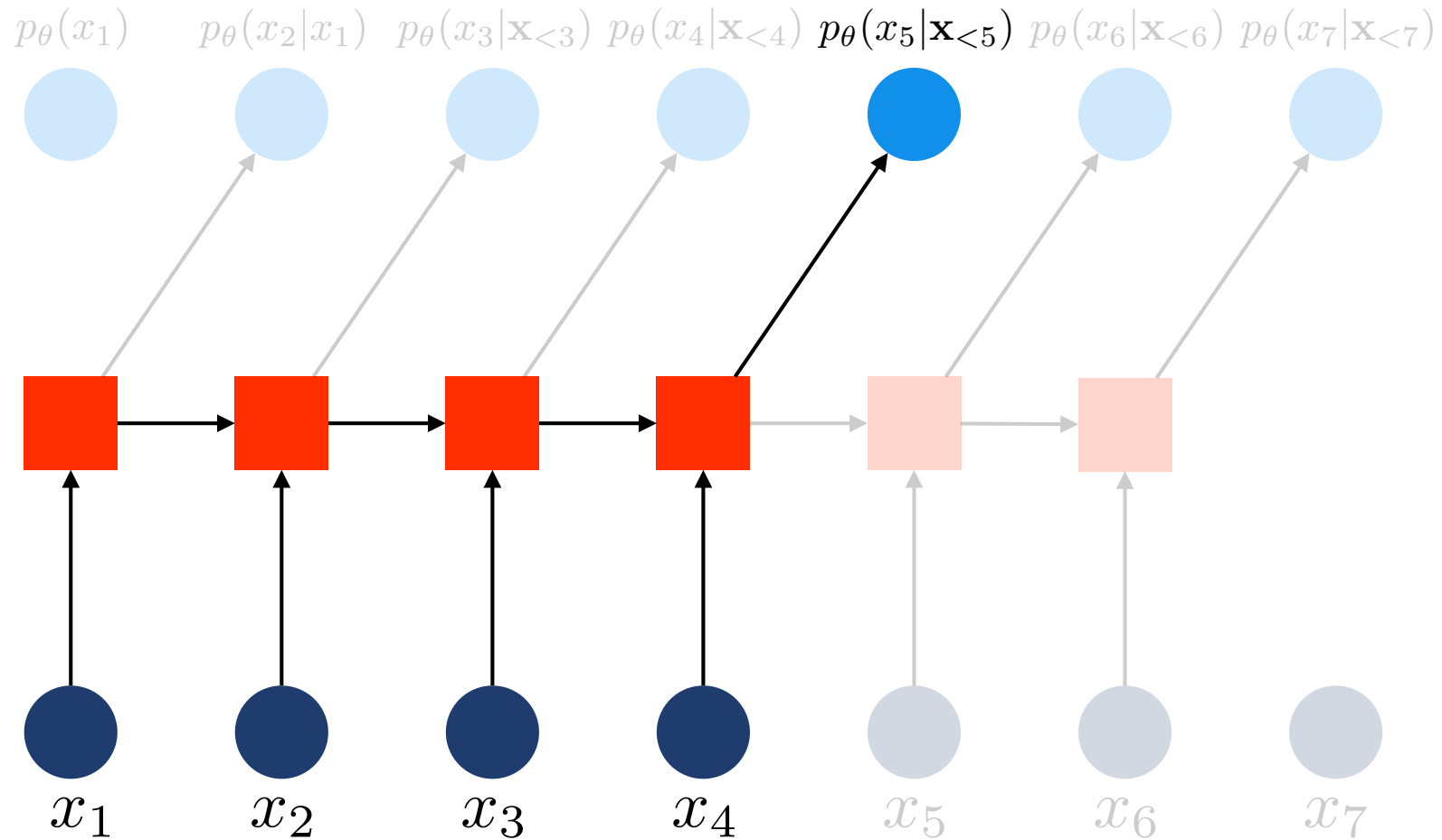


see **Deep Learning** (Chapter 10), *Goodfellow et al.*, 2016



# models

can parameterize conditional distributions using a **recurrent neural network**

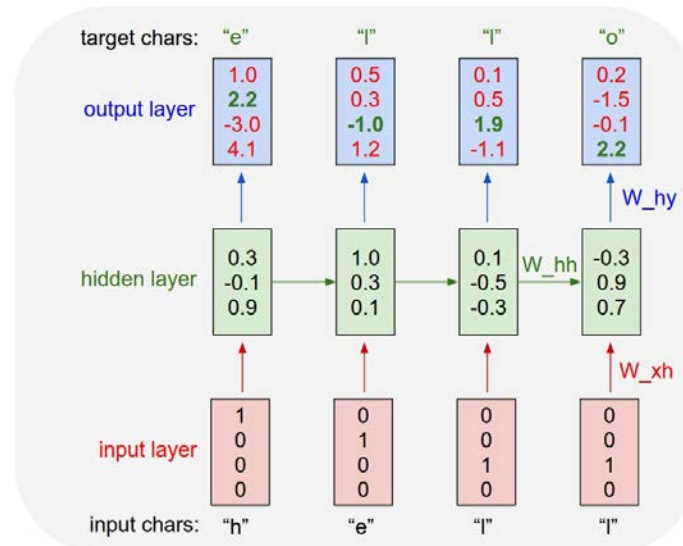


see **Deep Learning** (Chapter 10), *Goodfellow et al.*, 2016

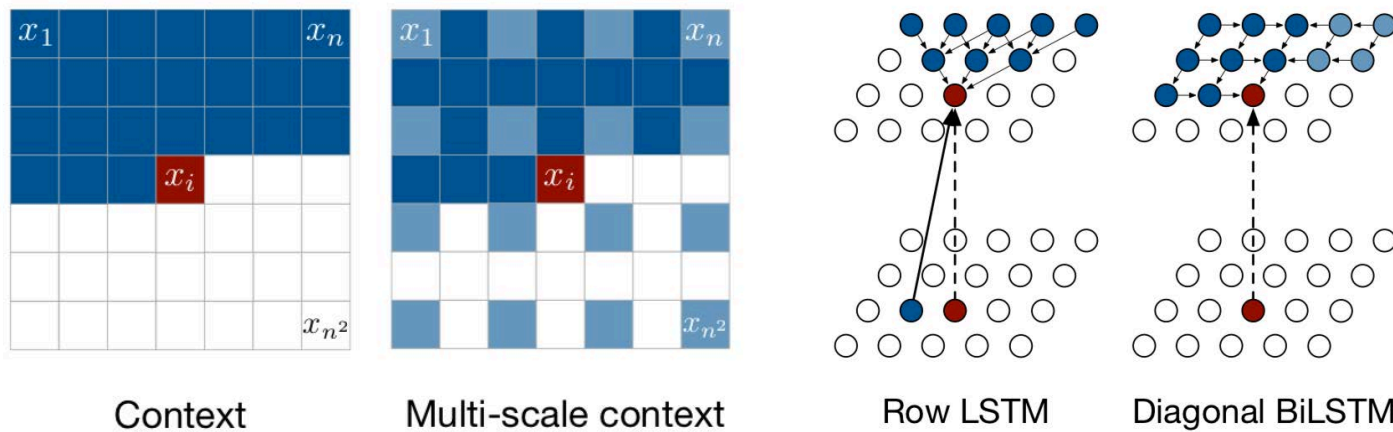


# models

can parameterize conditional distributions using a **recurrent neural network**



The Unreasonable Effectiveness of Recurrent Neural Networks, Karpathy, 2015

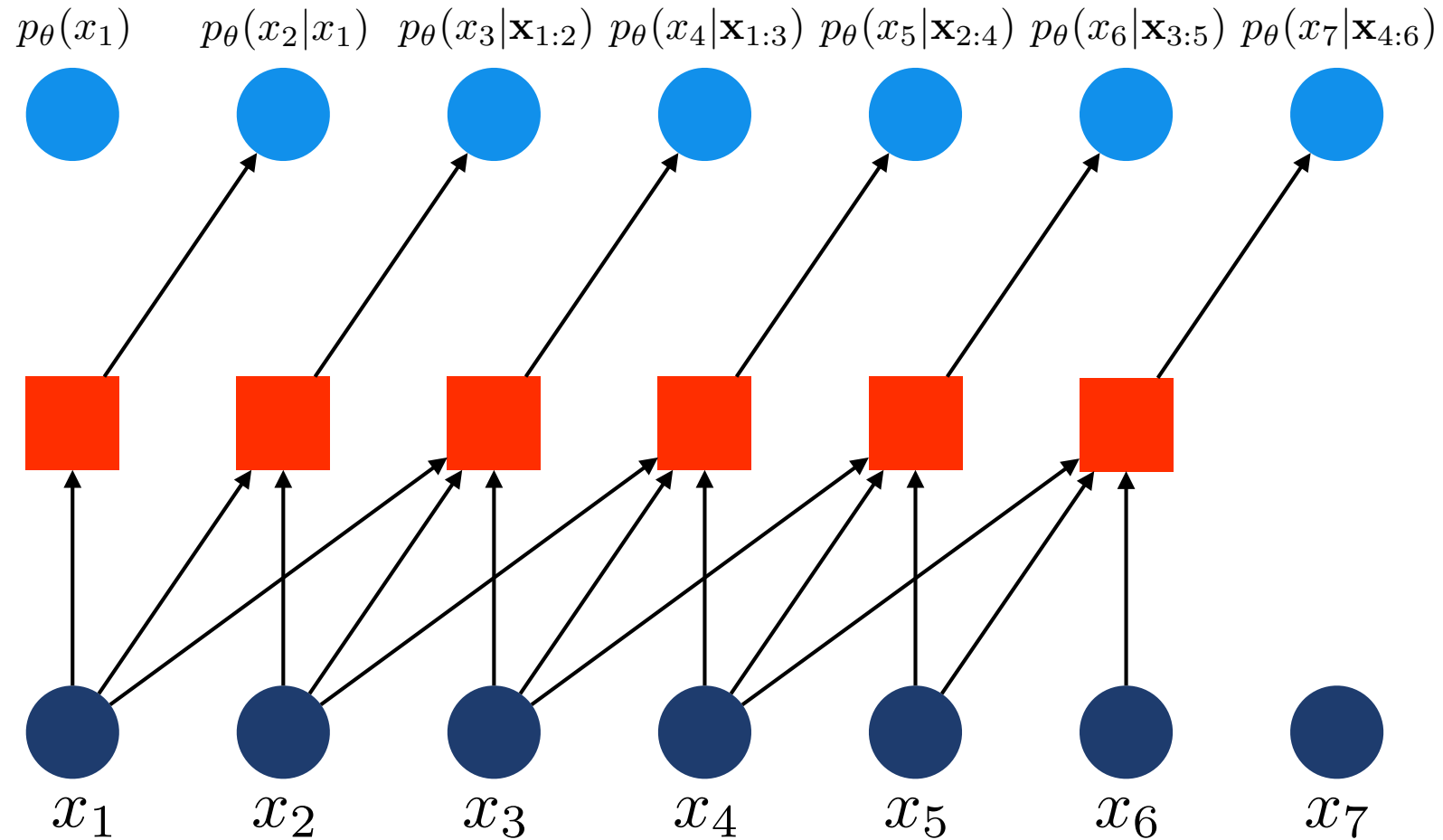


Pixel Recurrent Neural Networks, van den Oord et al., 2016



# models

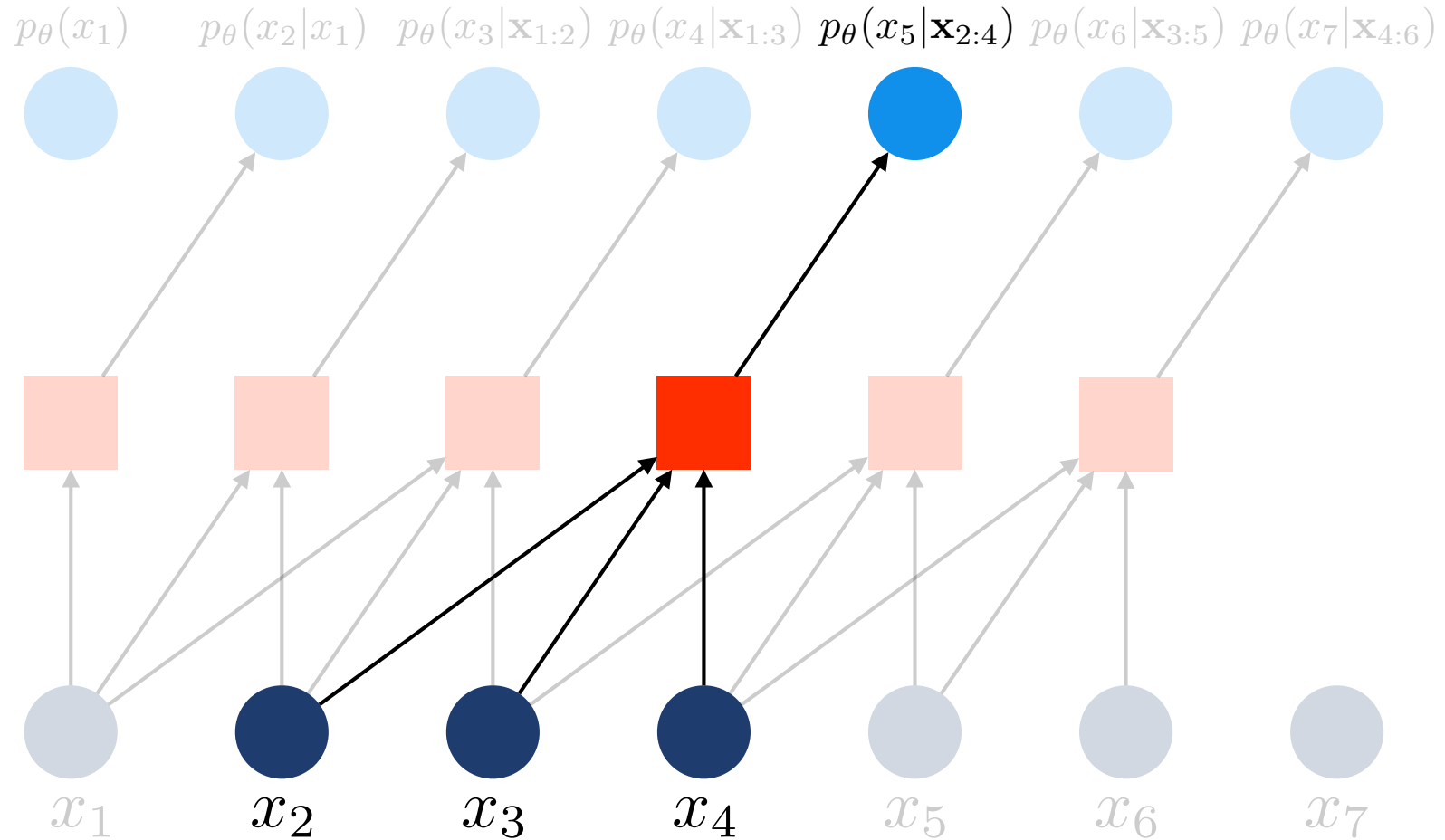
can condition on a local window using **convolutional neural networks**





# models

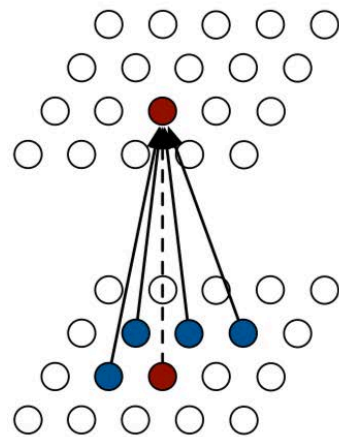
can condition on a local window using **convolutional neural networks**





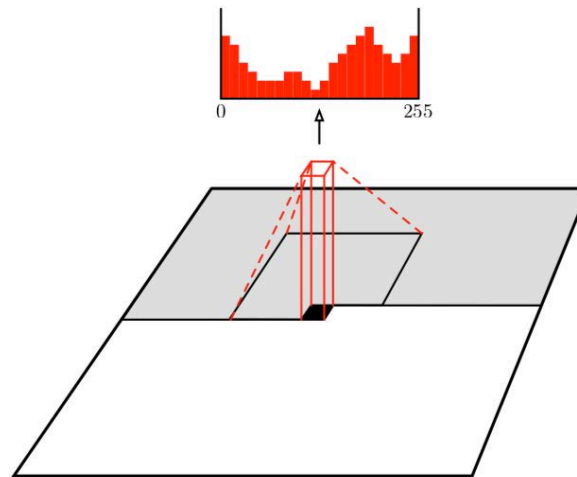
# models

can condition on a local window using **convolutional neural networks**

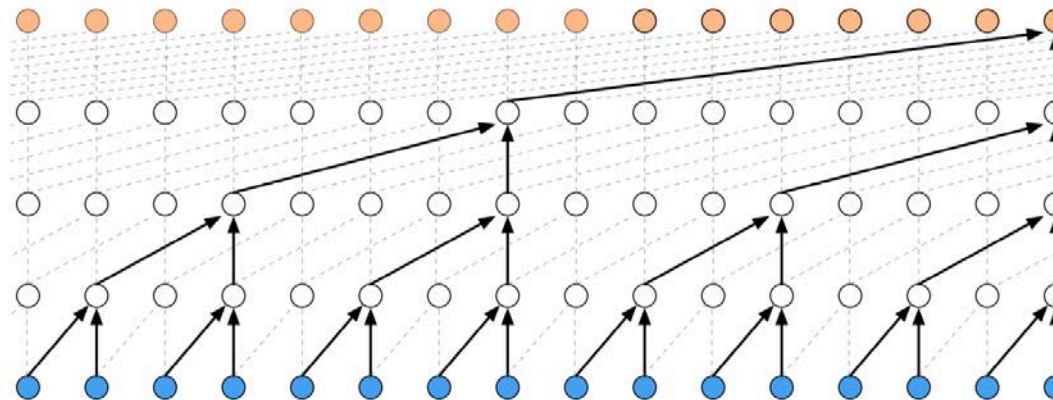
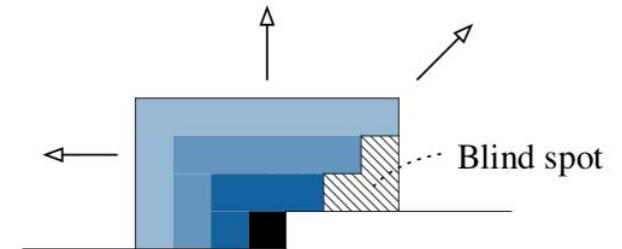


PixelCNN

Pixel Recurrent Neural Networks,  
*van den Oord et al., 2016*



Conditional Image Generation with PixelCNN Decoders,  
*van den Oord et al., 2016*



WaveNet: A Generative Model for Raw Audio, *van den Oord et al., 2016*

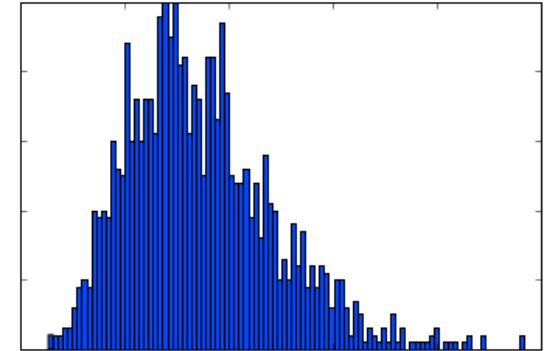


## output distributions

need to choose a form for the conditional **output distribution**,  
i.e. how do we express  $p(x_j|x_1, \dots, x_{j-1})$ ?

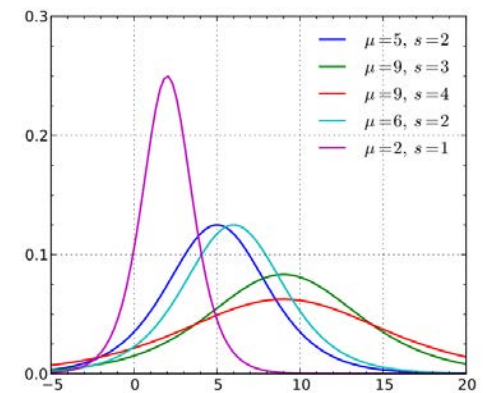
model the data as **discrete** variables

—————→ categorical output



model the data as **continuous** variables

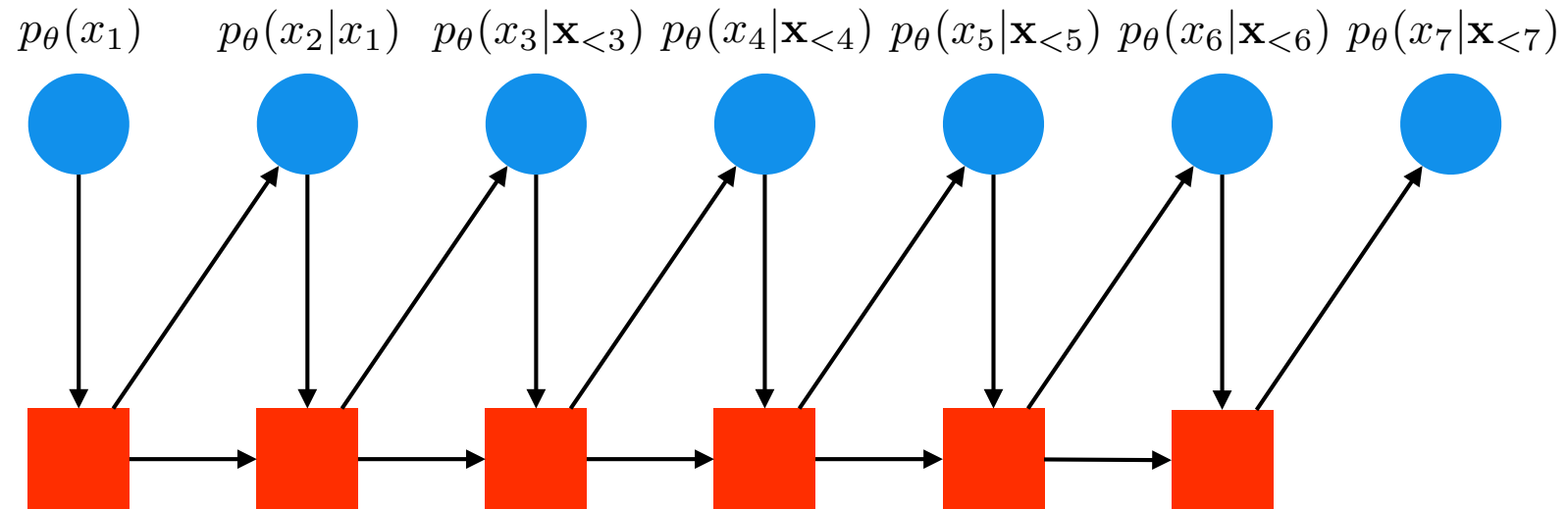
—————→ Gaussian, logistic, etc. output





# sampling

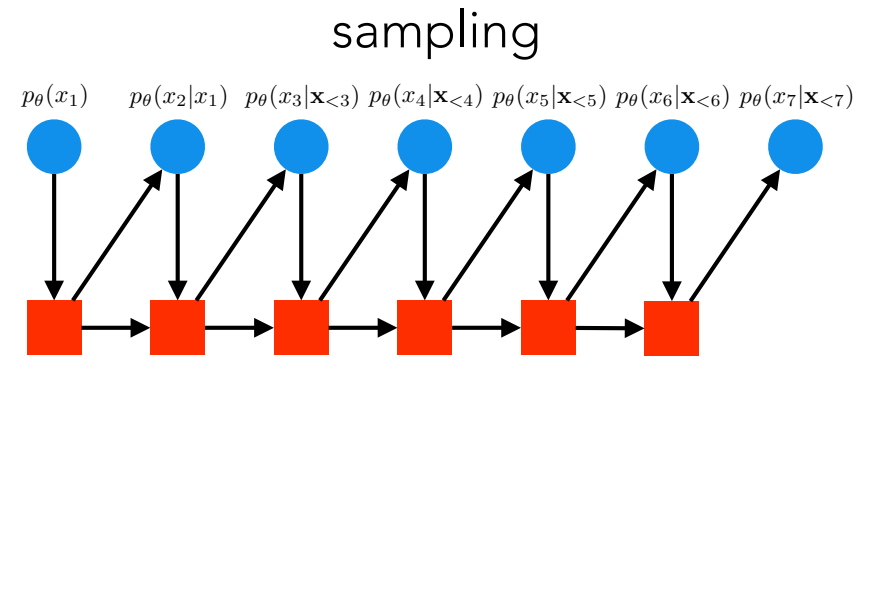
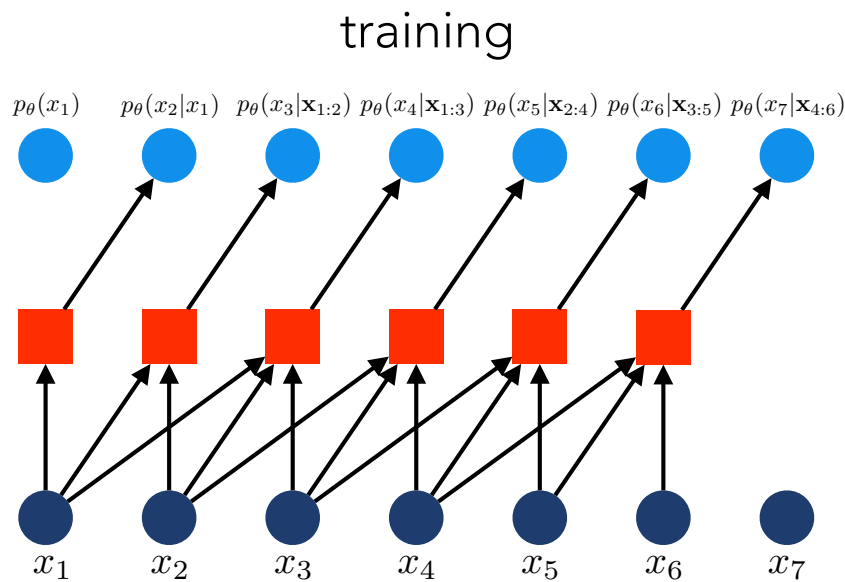
sample from the model by drawing from the output distribution





## question

what issues might arise with sampling from the model?



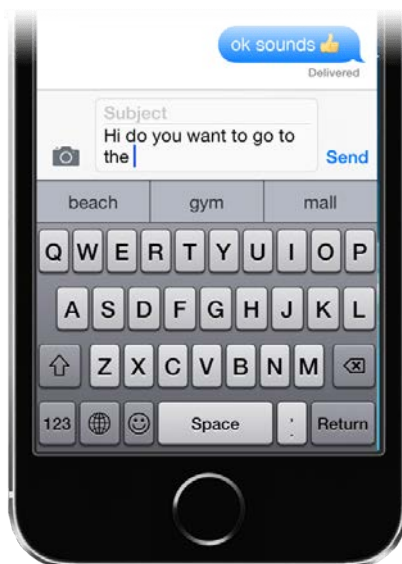
errors in the model distribution can accumulate, leading to poor samples

see teacher forcing



## example applications

text

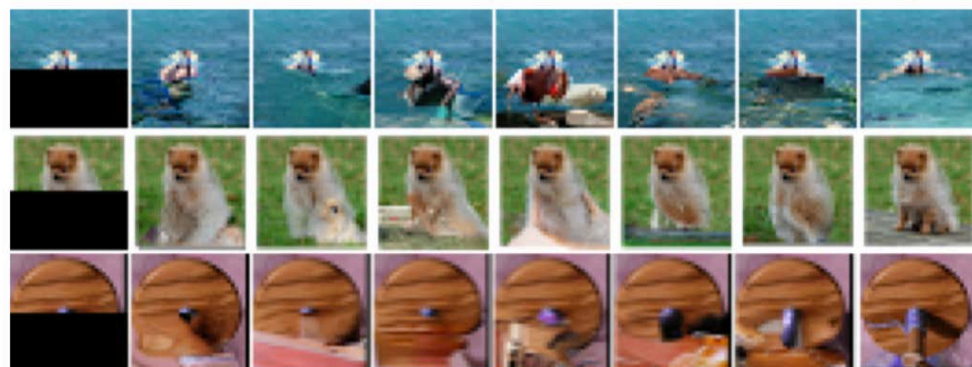


images

occluded

completions

original



**Pixel Recurrent Neural Networks**, *van den Oord et al.*, 2016

speech



1 Second



**WaveNet: A Generative Model for Raw Audio**, *van den Oord et al.*, 2016



SYSTEM PROMPT  
(HUMAN-WRITTEN)

*A train carriage containing controlled nuclear materials was stolen in Cincinnati today. Its whereabouts are unknown.*

MODEL COMPLETION  
(MACHINE-WRITTEN,  
FIRST TRY)

The incident occurred on the downtown train line, which runs from Covington and Ashland stations.

In an email to Ohio news outlets, the U.S. Department of Energy said it is working with the Federal Railroad Administration to find the thief.

"The theft of this nuclear material will have significant negative consequences on public and environmental health, our workforce and the economy of our nation," said Tom Hicks, the U.S. Energy Secretary, in a statement. "Our top priority is to secure the theft and ensure it doesn't happen again."

The stolen material was taken from the University of Cincinnati's Research Triangle Park nuclear research site, according to a news release from Department officials.

The Nuclear Regulatory Commission did not immediately release any information.

According to the release, the U.S. Department of Energy's Office of Nuclear Material Safety and Security is leading that team's investigation.

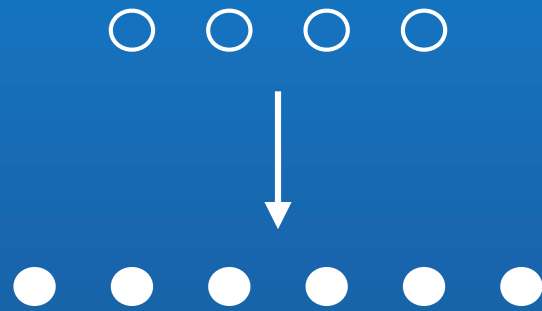
"The safety of people, the environment and the nation's nuclear stockpile is our highest priority," Hicks said. "We will get to the bottom of this and make no excuses."

Attention is All You Need, Vaswani et al., 2017

Improving Language Understanding by Generative Pre-Training, Radford et al., 2018

Language Models as Unsupervised Multi-task Learners, Radford et al., 2019

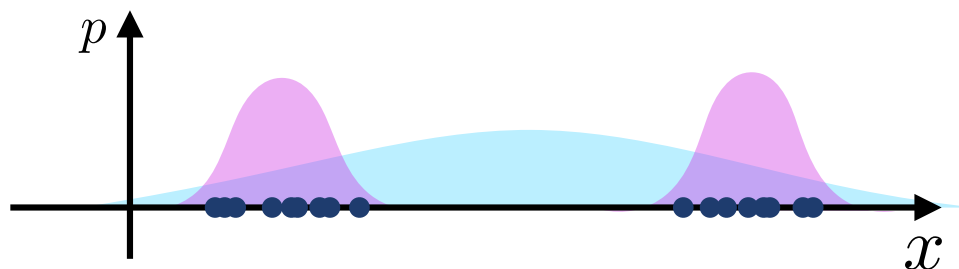




*explicit*  
*latent variable models*



latent variables result in mixtures of distributions



### approach 1

*directly fit a distribution to the data*

$$p_{\theta}(x) = \mathcal{N}(x; \mu, \sigma^2)$$

### approach 2

*use a latent variable to model the data*

$$p_{\theta}(x, z) = p_{\theta}(x|z)p_{\theta}(z) = \mathcal{N}(x; \mu_x(z), \sigma_x^2(z))\mathcal{B}(z; \mu_z)$$

$$p_{\theta}(x) = \sum_z p_{\theta}(x, z)$$

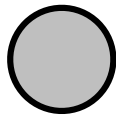
$$= \underbrace{\mu_z \cdot \mathcal{N}(x; \mu_x(1), \sigma_x^2(1))}_{\text{mixture component}} + \underbrace{(1 - \mu_z) \cdot \mathcal{N}(x; \mu_x(0), \sigma_x^2(0))}_{\text{mixture component}}$$



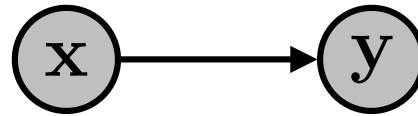
probabilistic graphical models provide a framework for modeling relationships between random variables

### PLATE NOTATION

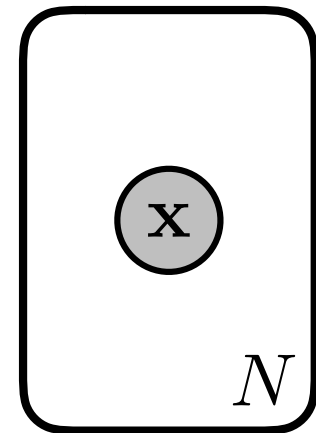
observed variable



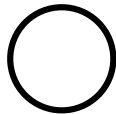
directed



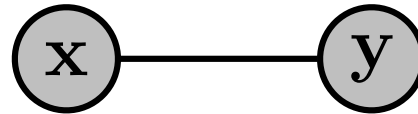
set of variables



unobserved (latent)  
variable



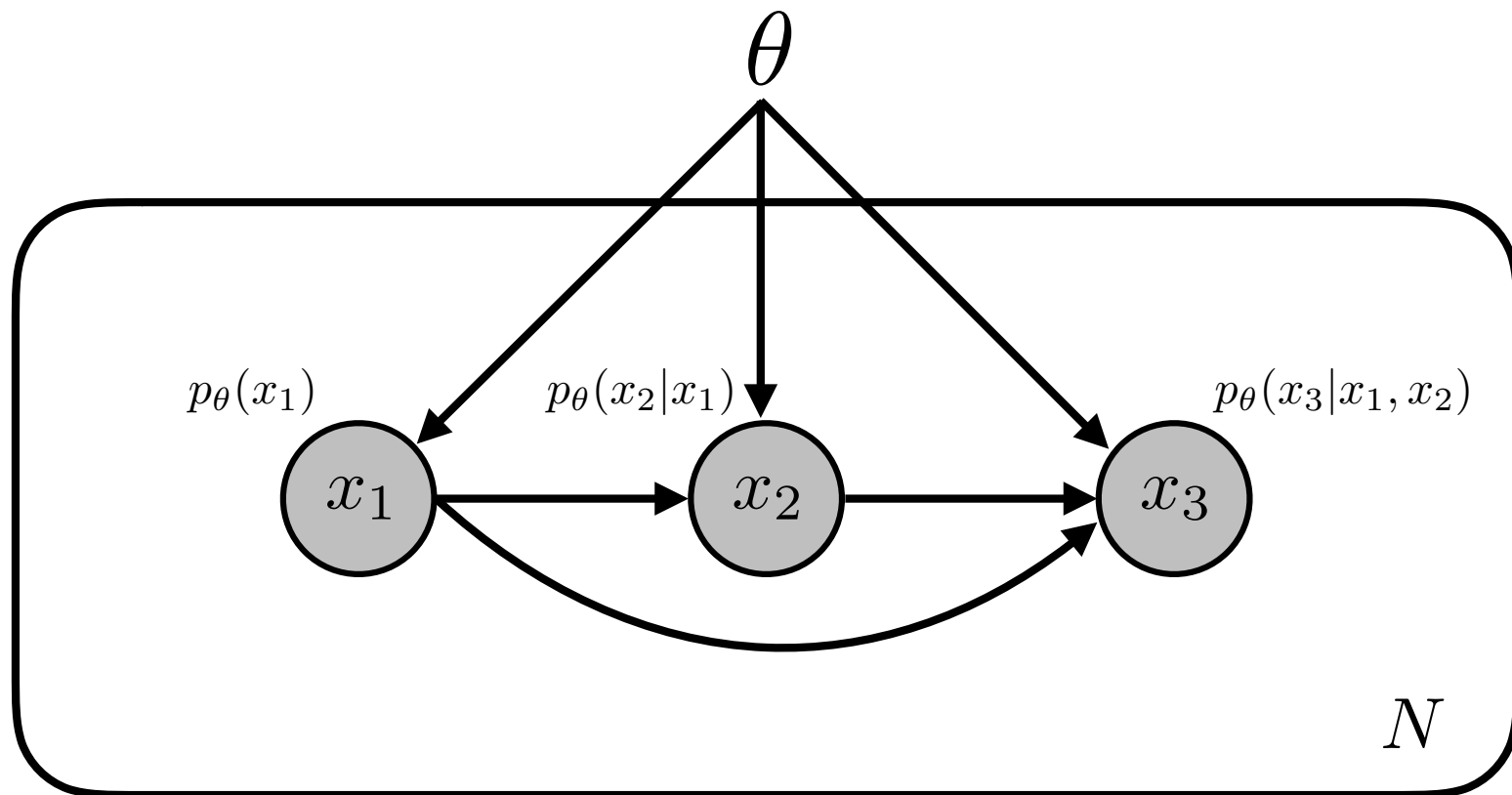
undirected





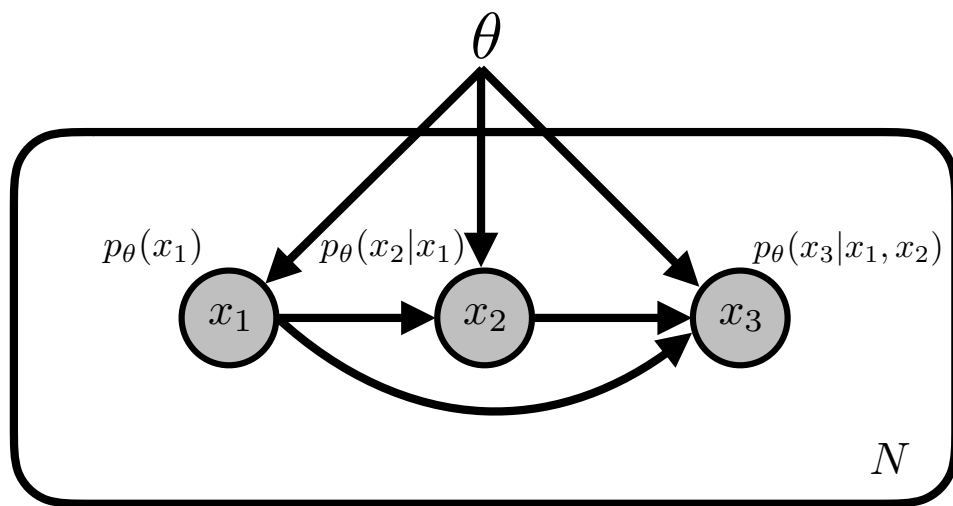
question

represent an auto-regressive model of 3 random variables  
with plate notation

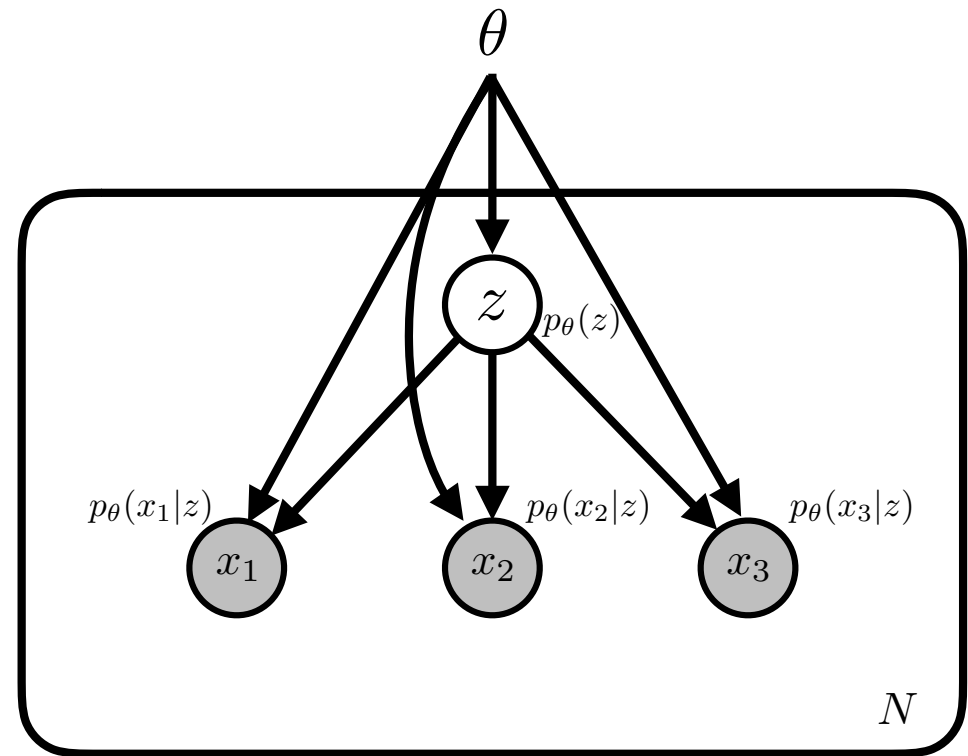




comparing *auto-regressive models* and *latent variable models*



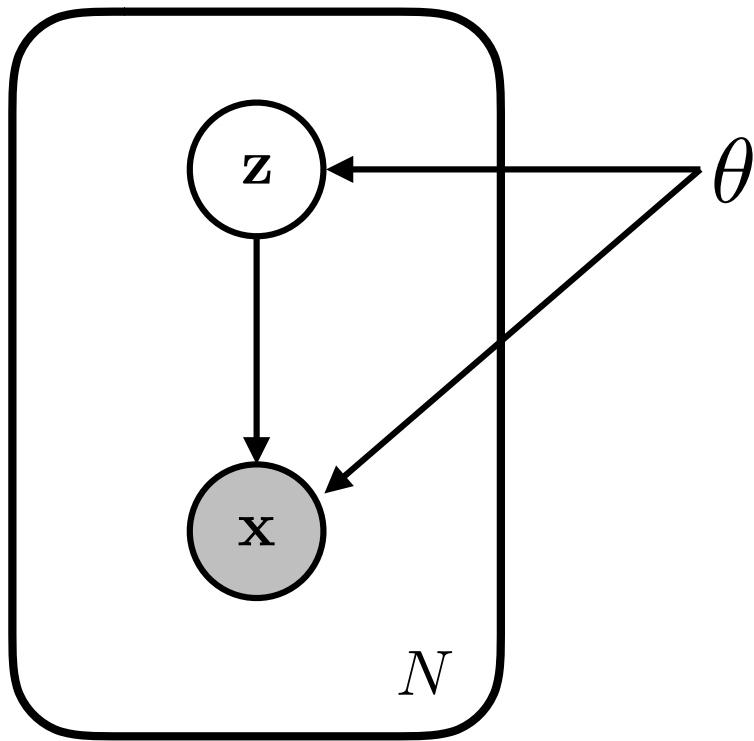
auto-regressive model



latent variable model



## directed latent variable model



Generation

**GENERATIVE MODEL**

$$\overset{\text{joint}}{\curvearrowright} p(\mathbf{x}, \mathbf{z}) = p(\mathbf{x}|\mathbf{z}) p(\mathbf{z}) \overset{\text{prior}}{\curvearrowleft}$$

$\uparrow$   
conditional likelihood

1. sample  $\mathbf{z}$  from  $p(\mathbf{z})$
2. use  $\mathbf{z}$  samples to sample  $\mathbf{x}$  from  $p(\mathbf{x}|\mathbf{z})$

*intuitive example: graphics engine*

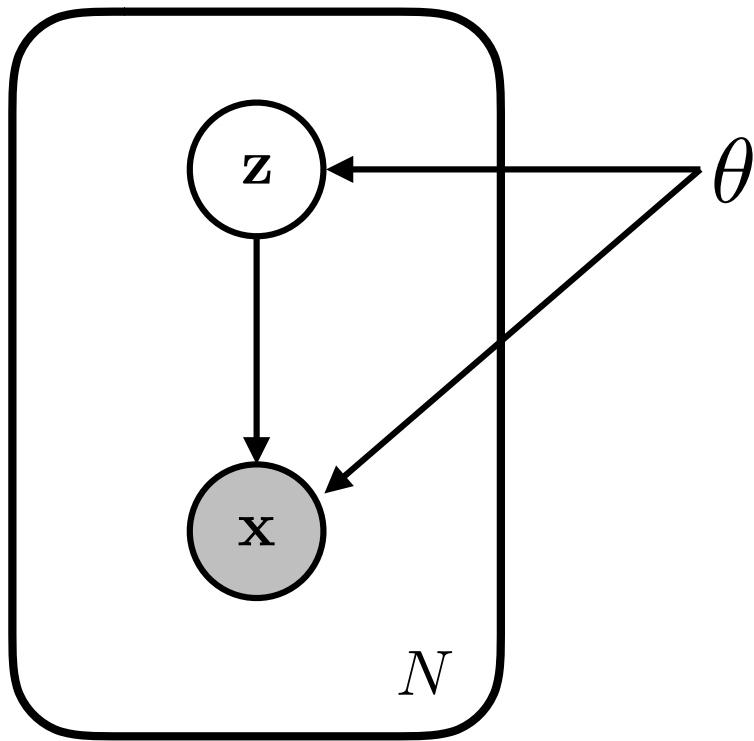
object  $\sim p(\text{objects})$   
lighting  $\sim p(\text{lighting})$   
background  $\sim p(\text{bg})$

RENDER





# directed latent variable model



## Posterior Inference

**INFERENCE**

$$p(\mathbf{z}|\mathbf{x}) = \frac{p(\mathbf{x}, \mathbf{z})}{p(\mathbf{x})}$$

posterior

joint

marginal likelihood

use Bayes' rule

provides conditional distribution  
over latent variables

*intuitive example*



observation

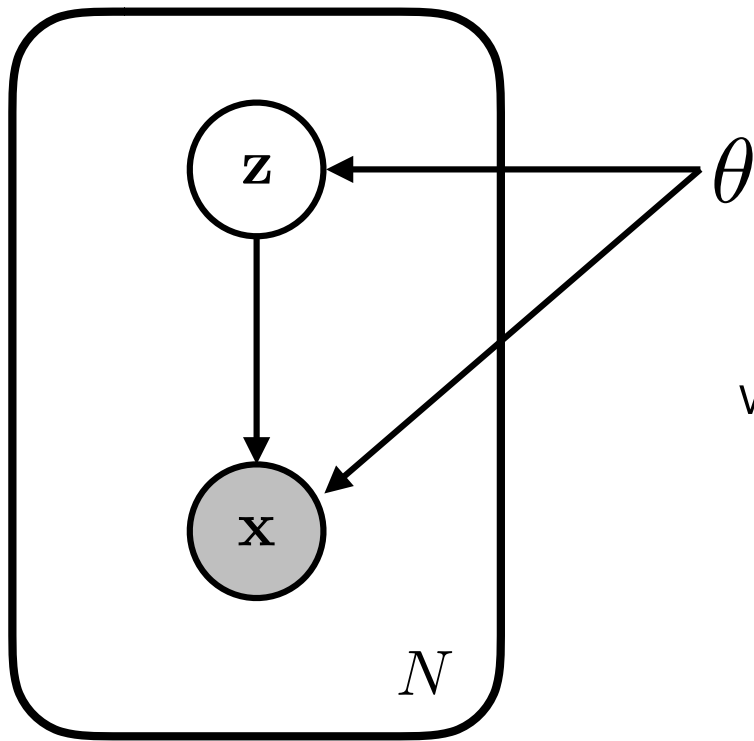
what is the probability that I am observing a cat  
given these pixel observations?

$$p(\text{cat} | \text{image}) = \frac{p(\text{image} | \text{cat}) p(\text{cat})}{p(\text{image})}$$



# directed latent variable model

## Model Evaluation



### MARGINALIZATION

marginal likelihood  $p(\mathbf{x}) = \int p(\mathbf{x}, \mathbf{z}) d\mathbf{z}$  joint

to evaluate the likelihood of an observation, we need to *marginalize* over all latent variables

i.e. consider all possible underlying states

### intuitive example



observation

how likely is this observation under my model?  
(what is the probability of observing this?)

for all objects, lighting, backgrounds, etc.:  
how plausible is this example?



# maximum likelihood estimation

*maximize the log-likelihood (under the model) of the true data examples*

$$\theta^* = \arg \max_{\theta} \mathbb{E}_{p_{\text{data}}(\mathbf{x})} [\log p_{\theta}(\mathbf{x})] \approx \frac{1}{N} \sum_{i=1}^N \log p_{\theta}(\mathbf{x}^{(i)})$$

for latent variable models:

*discrete*

$$\log p_{\theta}(\mathbf{x}) = \log \sum_z p_{\theta}(\mathbf{x}, \mathbf{z})$$

or

*continuous*

$$\log p_{\theta}(\mathbf{x}) = \log \int p_{\theta}(\mathbf{x}, \mathbf{z}) d\mathbf{z}$$

marginalizing is often intractable in practice



# variational inference

*lower bound the log-likelihood by introducing an approximate posterior*

introduce an **approximate posterior**  $q(\mathbf{z}|\mathbf{x})$

$$\log p_{\theta}(\mathbf{x}) = \mathcal{L}(\mathbf{x}) + D_{KL}(q(\mathbf{z}|\mathbf{x})||p_{\theta}(\mathbf{z}|\mathbf{x}))$$

$$\text{where } \mathcal{L}(\mathbf{x}) = \mathbb{E}_{q(\mathbf{z}|\mathbf{x})} [\log p_{\theta}(\mathbf{x}, \mathbf{z}) - \log q(\mathbf{z}|\mathbf{x})]$$

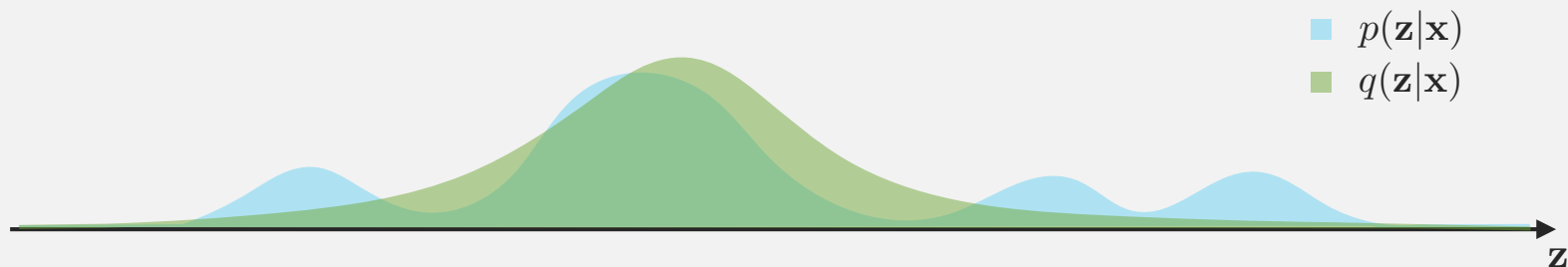
$$D_{KL} \geq 0 \longrightarrow \mathcal{L}(\mathbf{x}) \leq \log p_{\theta}(\mathbf{x}) \quad (\text{lower bound})$$

## variational expectation maximization (EM)

E-Step: optimize  $\mathcal{L}(\mathbf{x})$  w.r.t.  $q(\mathbf{z}|\mathbf{x})$

M-Step: optimize  $\mathcal{L}(\mathbf{x})$  w.r.t.  $\theta$

the E-Step indirectly minimizes  $D_{KL}(q(\mathbf{z}|\mathbf{x})||p_{\theta}(\mathbf{z}|\mathbf{x}))$





## interpreting the lower bound

we can write the lower bound as

$$\begin{aligned}\mathcal{L} &\equiv \mathbb{E}_{\mathbf{z} \sim q(\mathbf{z}|\mathbf{x})} [\log p(\mathbf{x}, \mathbf{z}) - \log q(\mathbf{z}|\mathbf{x})] \\ &= \mathbb{E}_{\mathbf{z} \sim q(\mathbf{z}|\mathbf{x})} [\log p(\mathbf{x}|\mathbf{z})p(\mathbf{z}) - \log q(\mathbf{z}|\mathbf{x})] \\ &= \mathbb{E}_{\mathbf{z} \sim q(\mathbf{z}|\mathbf{x})} [\log p(\mathbf{x}|\mathbf{z}) + \log p(\mathbf{z}) - \log q(\mathbf{z}|\mathbf{x})] \\ &= \underbrace{\mathbb{E}_{\mathbf{z} \sim q(\mathbf{z}|\mathbf{x})} [\log p(\mathbf{x}|\mathbf{z})]}_{\text{reconstruction}} - \underbrace{D_{KL}(q(\mathbf{z}|\mathbf{x})||p(\mathbf{z}))}_{\text{regularization}}\end{aligned}$$

$q(\mathbf{z}|\mathbf{x})$  is optimized to represent the data while staying close to the prior

connections to *compression, information theory*



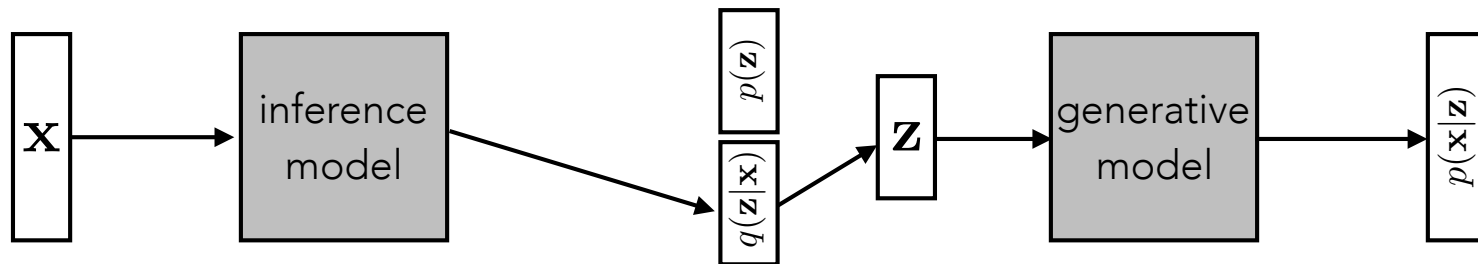
# variational autoencoder (VAE)

## **variational expectation maximization (EM)**

E-Step: optimize  $\mathcal{L}(\mathbf{x})$  w.r.t.  $q(\mathbf{z}|\mathbf{x})$

M-Step: optimize  $\mathcal{L}(\mathbf{x})$  w.r.t.  $\theta$

use a separate **inference model** to directly output approximate posterior estimates



learn both models jointly using stochastic backpropagation

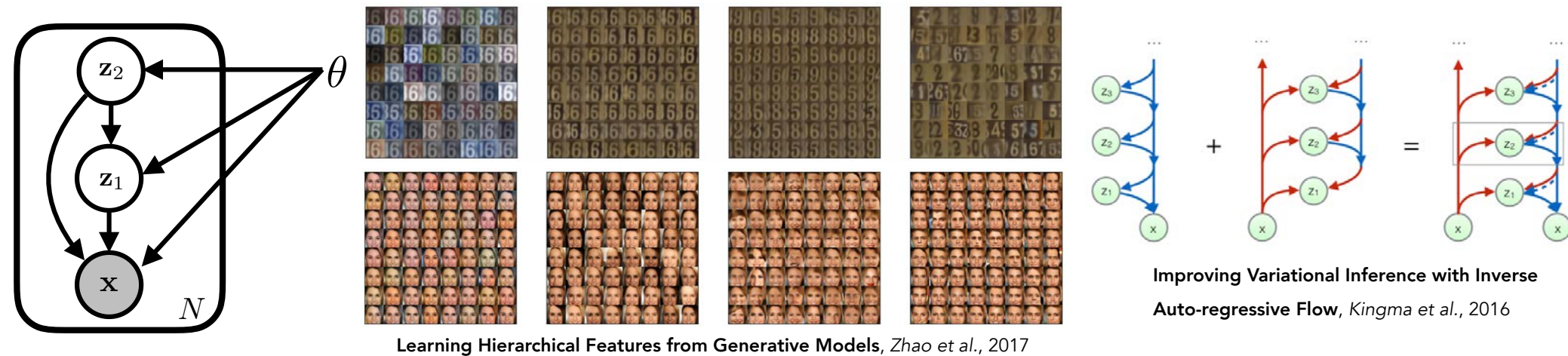
reparametrization trick:  $\mathbf{z} = \boldsymbol{\mu} + \boldsymbol{\sigma} \odot \boldsymbol{\epsilon}$        $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

**Autoencoding Variational Bayes**, Kingma & Welling, 2014

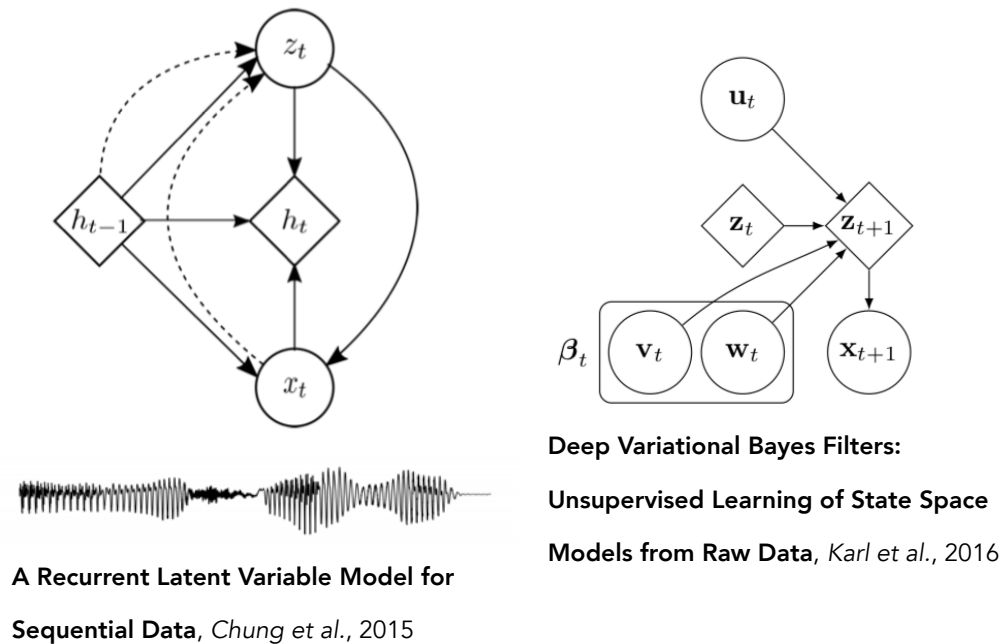
**Stochastic Backpropagation**, Rezende et al., 2014



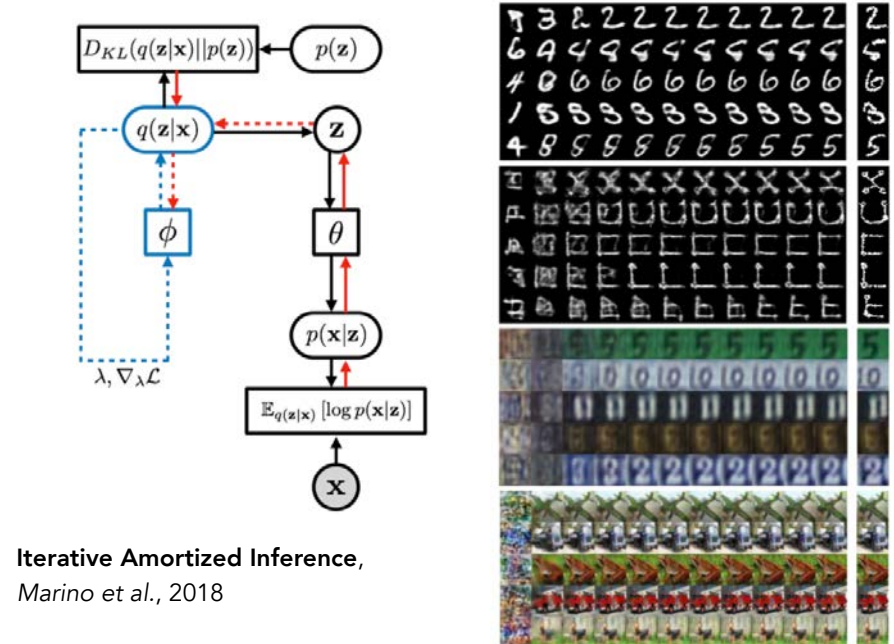
## hierarchical latent variable models



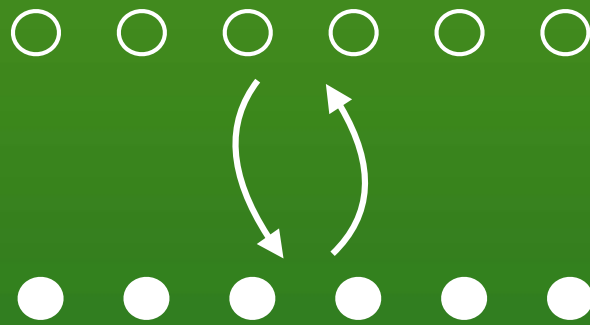
## sequential latent variable models



## iterative inference models







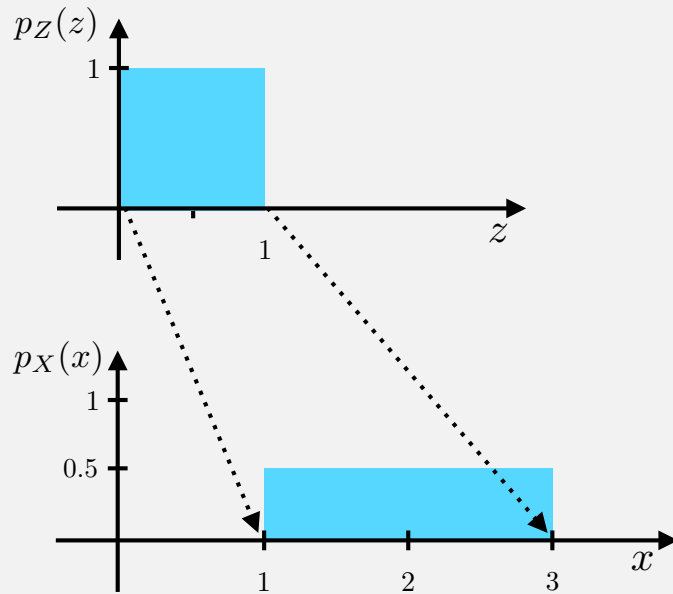
*invertible explicit  
latent variable models*



# change of variables

use an invertible mapping to directly evaluate the log likelihood

## simple example



sample  $z$  from a base distribution

$$z \sim p_Z(z) = \text{Uniform}(0, 1)$$

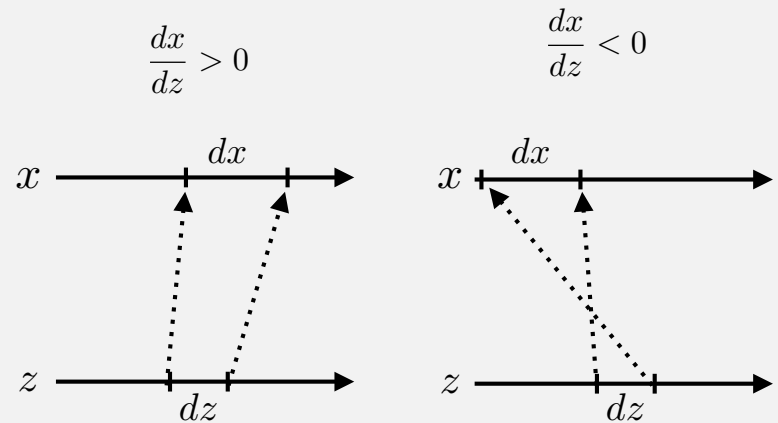
apply a transform to  $z$  to get a transformed distribution

$$x = f(z) = 2z + 1$$

$$p_X(x)dx = p_Z(z)dz$$

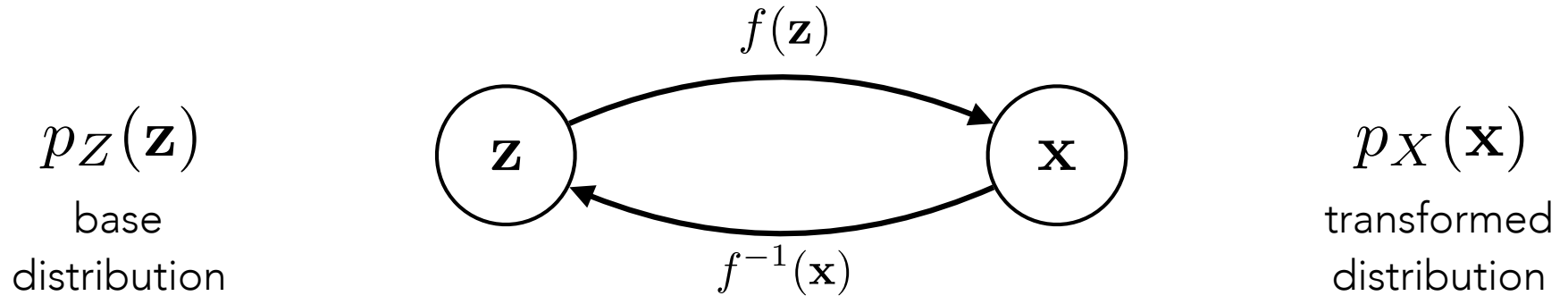
$$p_X(x) = p_Z(z) \left| \frac{dz}{dx} \right|$$

conservation of probability mass





## change of variables



### change of variables formula

$$p_X(\mathbf{x}) = p_Z(\mathbf{z}) \left| \det \mathbf{J}(f^{-1}(\mathbf{x})) \right|$$

or

$$\log p_X(\mathbf{x}) = \log p_Z(\mathbf{z}) + \log \left| \det \mathbf{J}(f^{-1}(\mathbf{x})) \right|$$

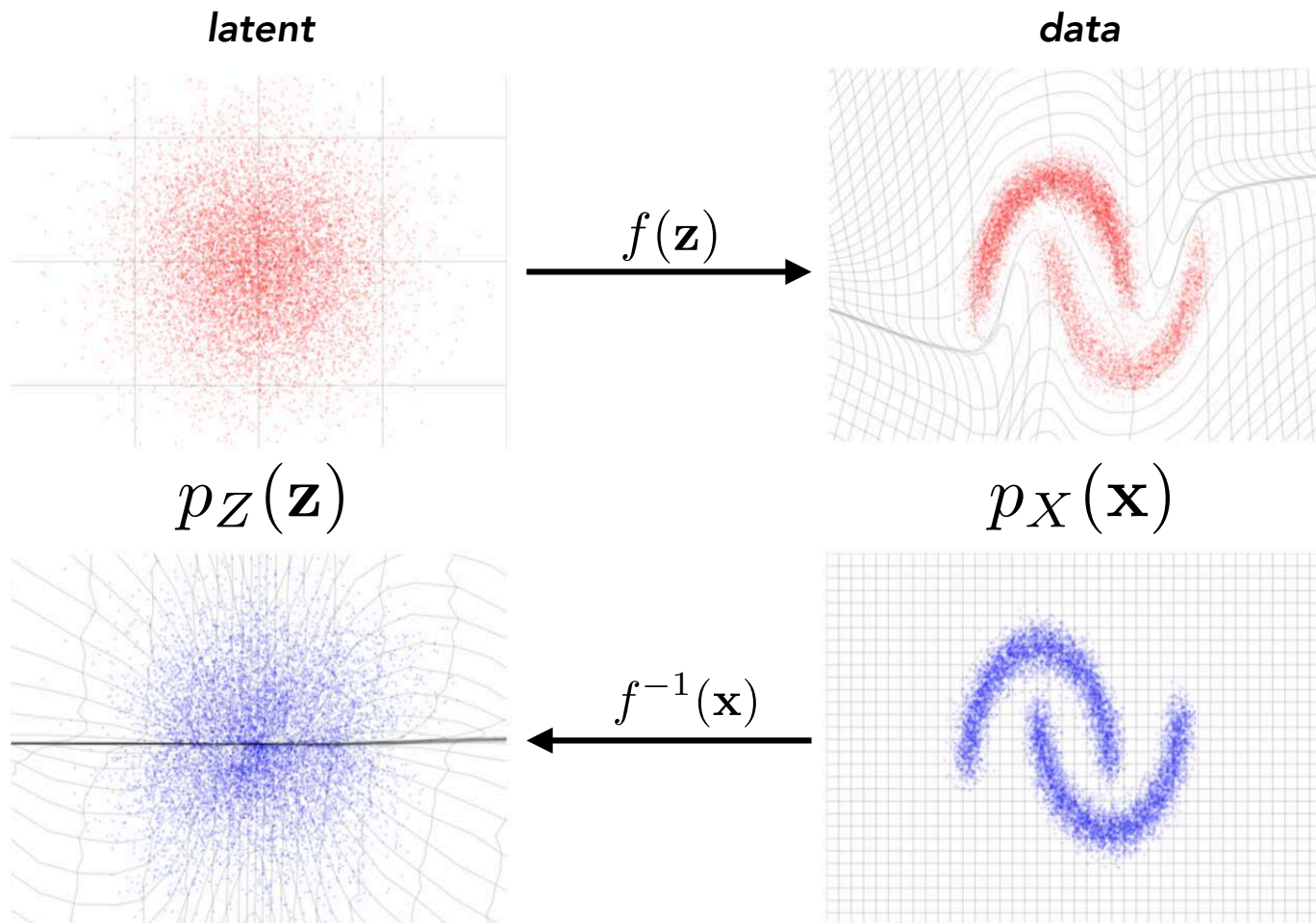
$\mathbf{J}(f^{-1}(\mathbf{x}))$  is the *Jacobian* matrix of the inverse transform

$\det \mathbf{J}(f^{-1}(\mathbf{x}))$  is the *local distortion in volume* from the transform



# change of variables

transform the data into a space that is easier to model





## maximum likelihood estimation

*maximize the log-likelihood (under the model) of the true data examples*

$$\theta^* = \arg \max_{\theta} \mathbb{E}_{p_{\text{data}}(\mathbf{x})} [\log p_{\theta}(\mathbf{x})] \approx \frac{1}{N} \sum_{i=1}^N \log p_{\theta}(\mathbf{x}^{(i)})$$

for invertible latent variable models:

$$\log p_{\theta}(\mathbf{x}) = \log p_{\theta}(\mathbf{z}) + \log |\det \mathbf{J}(f_{\theta}^{-1}(\mathbf{x}))|$$

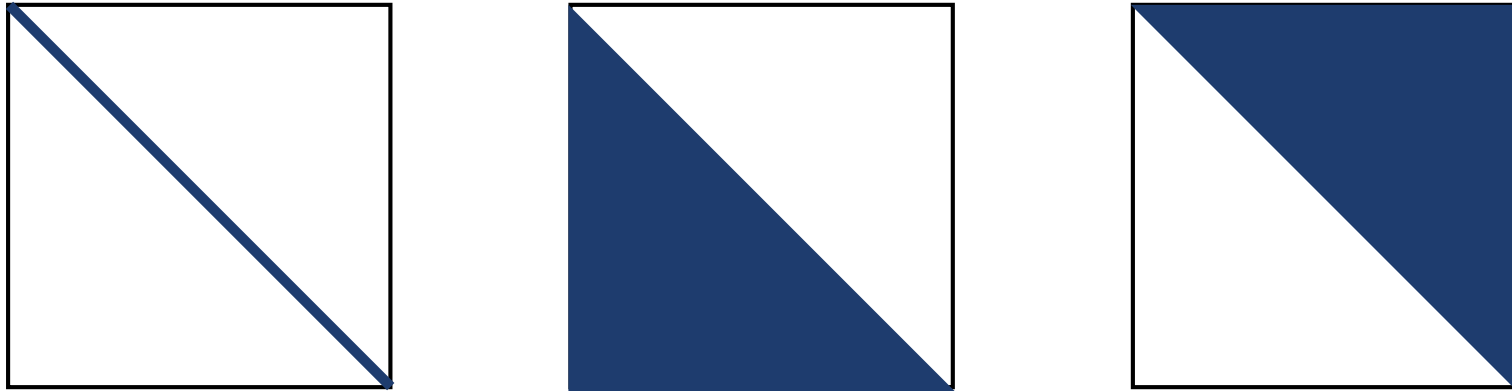
$$\theta^* = \arg \max_{\theta} \frac{1}{N} \sum_{i=1}^N \left[ \log p_{\theta}(\mathbf{z}^{(i)}) + \log |\det \mathbf{J}(f_{\theta}^{-1}(\mathbf{x}^{(i)}))| \right]$$



## change of variables

to use the change of variables formula, we need to evaluate  $\det \mathbf{J}(f^{-1}(\mathbf{x}))$

for an arbitrary  $N \times N$  Jacobian matrix, this is worst case  $O(N^3)$



restrict the transforms to those with diagonal or triangular inverse Jacobians

allows us to compute  $\det \mathbf{J}(f^{-1}(\mathbf{x}))$  in  $O(N)$

→ *product of diagonal entries*



# masked autoregressive flow (MAF)

*autoregressive sampling* can be interpreted as a transformed distribution

$$x_i \sim \mathcal{N}(x_i; \mu_i(\mathbf{x}_{1:i-1}), \sigma_i^2(\mathbf{x}_{1:i-1})) \longrightarrow x_i = \mu_i(\mathbf{x}_{1:i-1}) + \sigma_i(\mathbf{x}_{1:i-1}) \cdot z_i$$

where  $z_i \sim \mathcal{N}(z_i; 0, 1)$

must generate each  $x_i$  *sequentially*

however, we can parallelize the inverse transform:

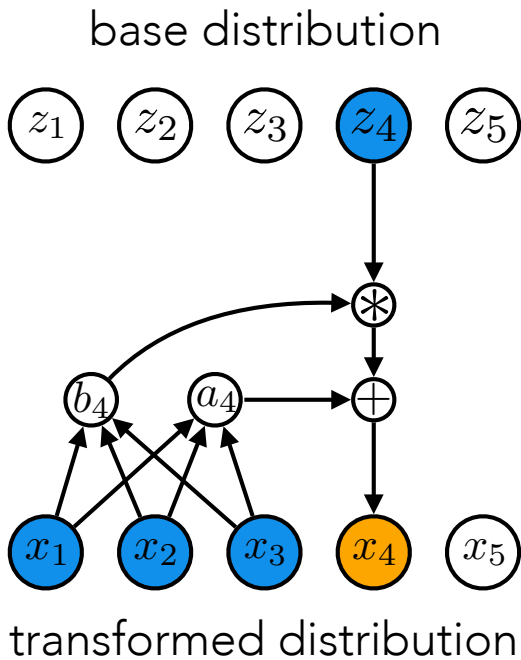
$$z_i = \frac{x_i - \mu_i(\mathbf{x}_{1:i-1})}{\sigma_i(\mathbf{x}_{1:i-1})}$$

**Masked Autoregressive Flow**, Papamakarios et al., 2017  
see also **Inverse Autoregressive Flow**, Kingma et al., 2016



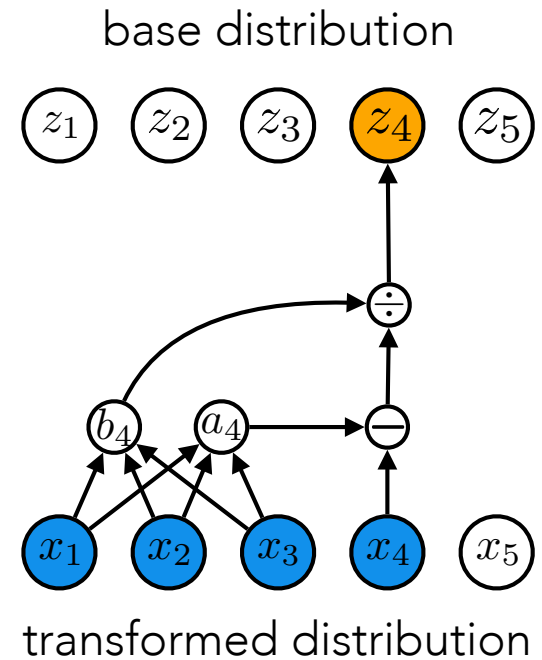
# masked autoregressive flow (MAF)

## TRANSFORM



$$x_4 = a_4(\mathbf{x}_{1:3}) + b_4(\mathbf{x}_{1:3}) \cdot z_4$$

## INVERSE TRANSFORM

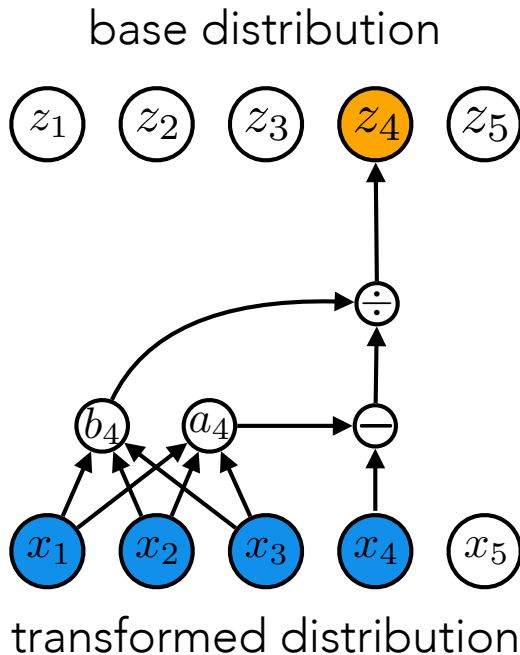


$$z_4 = \frac{x_4 - a_4(\mathbf{x}_{1:3})}{b_4(\mathbf{x}_{1:3})}$$



question

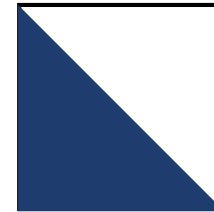
## INVERSE TRANSFORM



$$z_4 = \frac{x_4 - a_4(\mathbf{x}_{1:3})}{b_4(\mathbf{x}_{1:3})}$$

What is the form of  $\mathbf{J}(f^{-1}(\mathbf{x}))$ ?

lower triangular



each  $z_i$  only depends on  $\mathbf{x}_{1:i}$

What is  $\det \mathbf{J}(f^{-1}(\mathbf{x}))$ ?

product of diagonal elements of  $\mathbf{J}(f^{-1}(\mathbf{x}))$

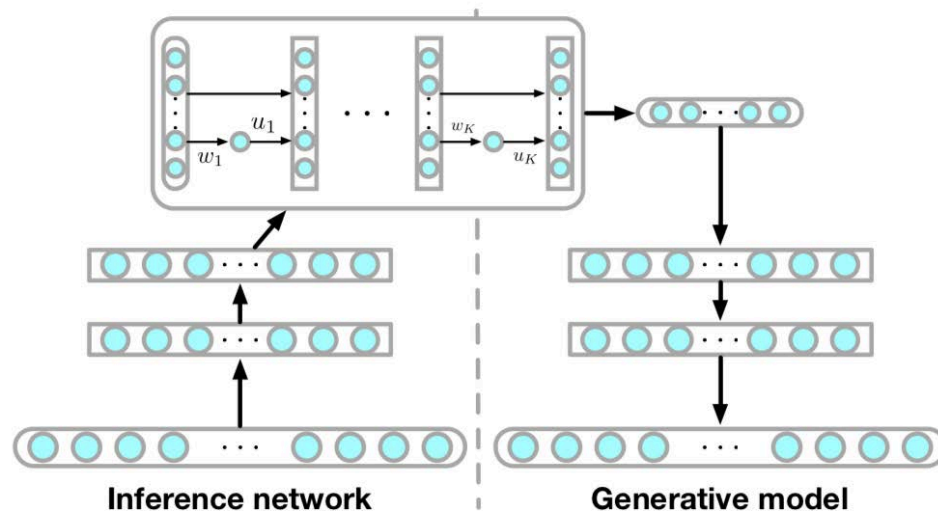
$$\det \mathbf{J}(f^{-1}(\mathbf{x})) = \prod_i \frac{1}{b_i(\mathbf{x}_{1:i})}$$



# normalizing flows (NF)

can also use the change of variables formula for variational inference

parameterize  $q(\mathbf{z}|\mathbf{x})$  as a transformed distribution

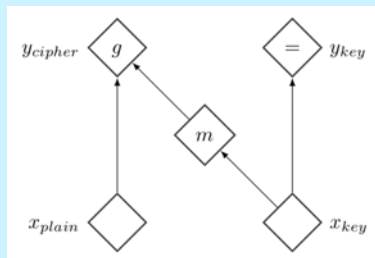


use more complex approximate posterior, but evaluate a simpler distribution

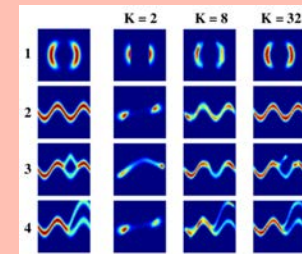
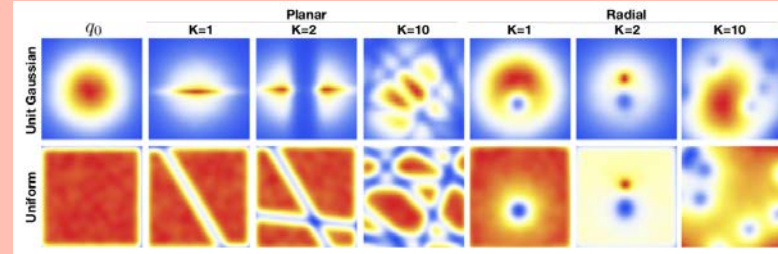
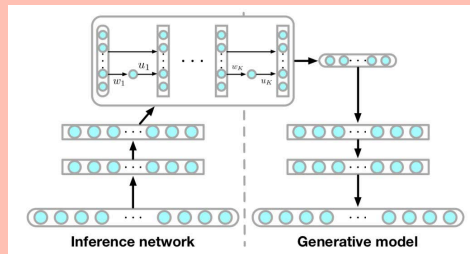


# some recent work

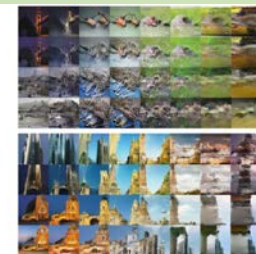
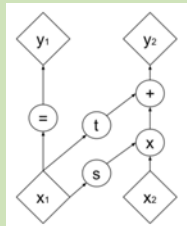
## NICE: Non-linear Independent Components Estimation, Dinh et al., 2014



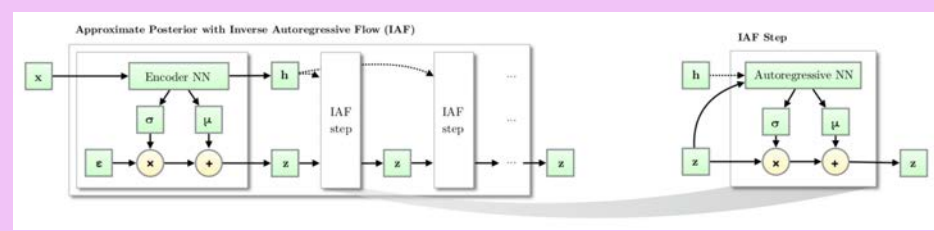
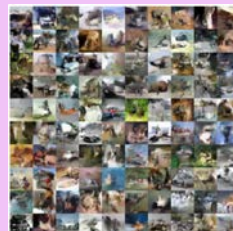
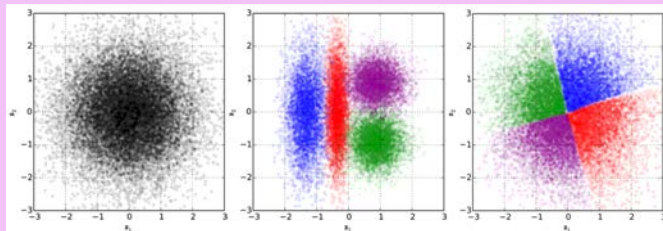
## Variational Inference with Normalizing Flows, Rezende & Mohamed, 2015



## Density Estimation Using Real NVP, Dinh et al., 2016



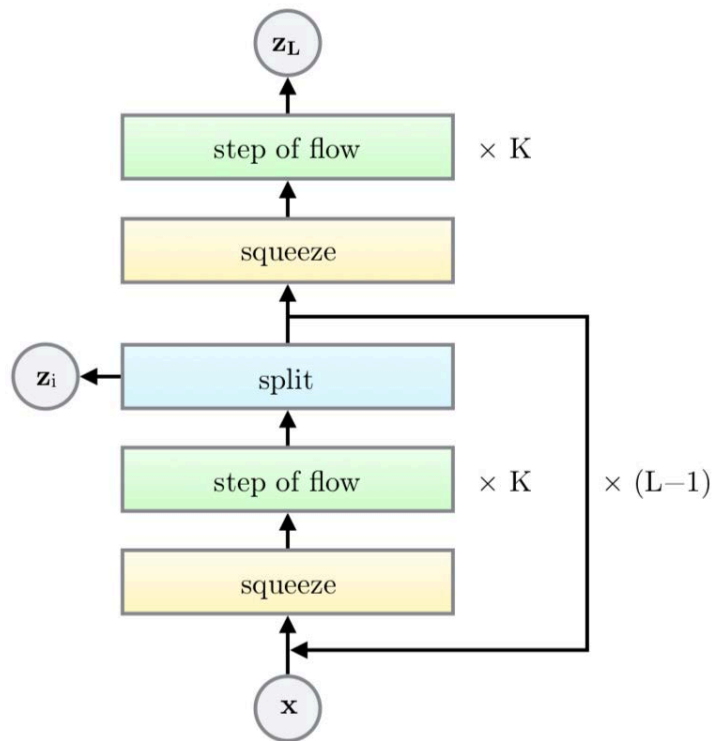
## Improving Variational Inference with Inverse Autoregressive Flow, Kingma et al., 2016





# Glow

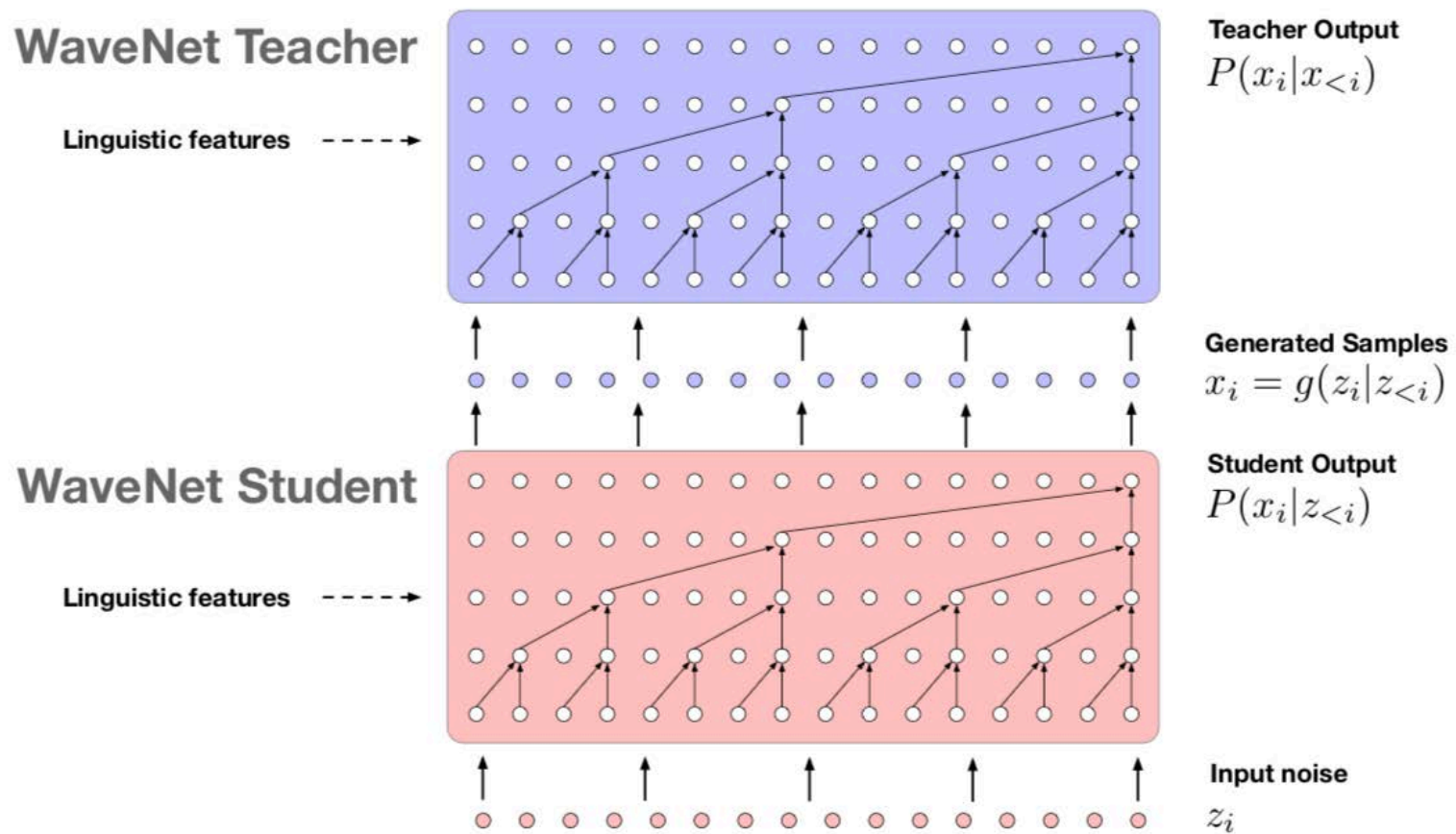
use  $1 \times 1$  convolutions to perform transform



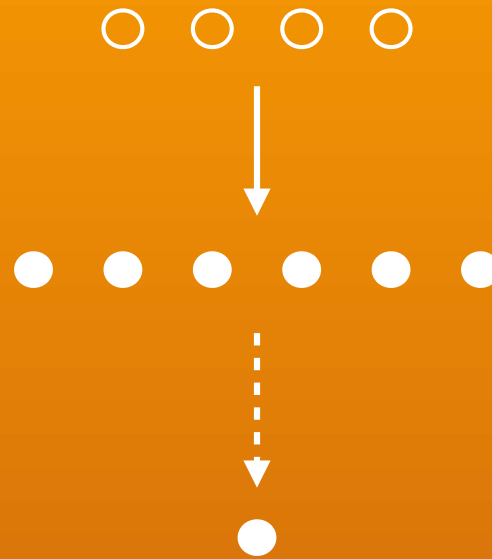


# Parallel WaveNet

*distill* an autoregressive distribution into a parallel transform



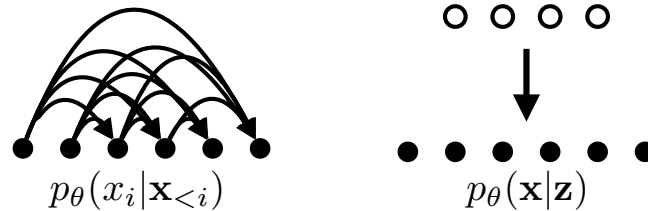




*implicit*  
*latent variable models*



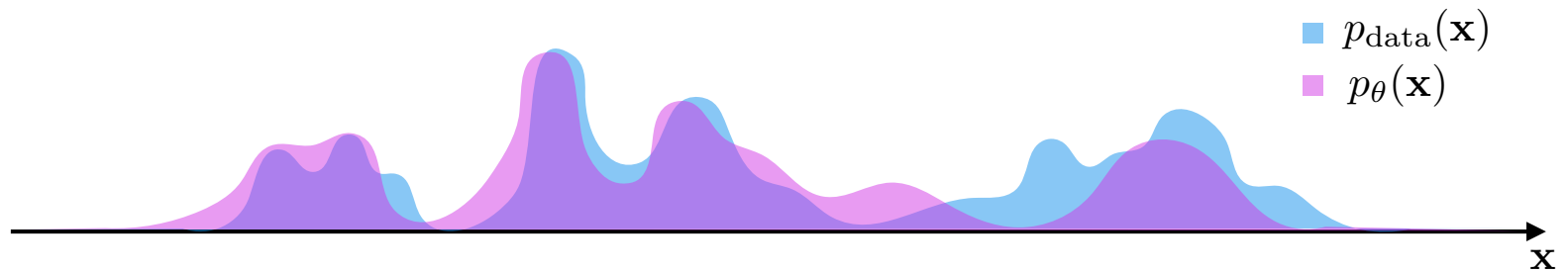
many generative models are defined in terms of an explicit likelihood  
in which  $p_{\theta}(\mathbf{x})$  has a parametric form



this may limit the types of distributions that can be learned



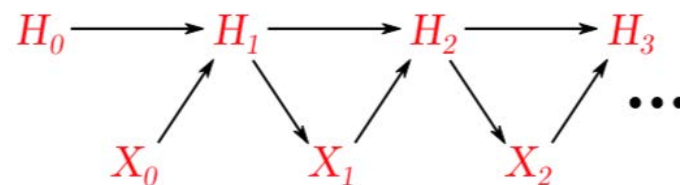
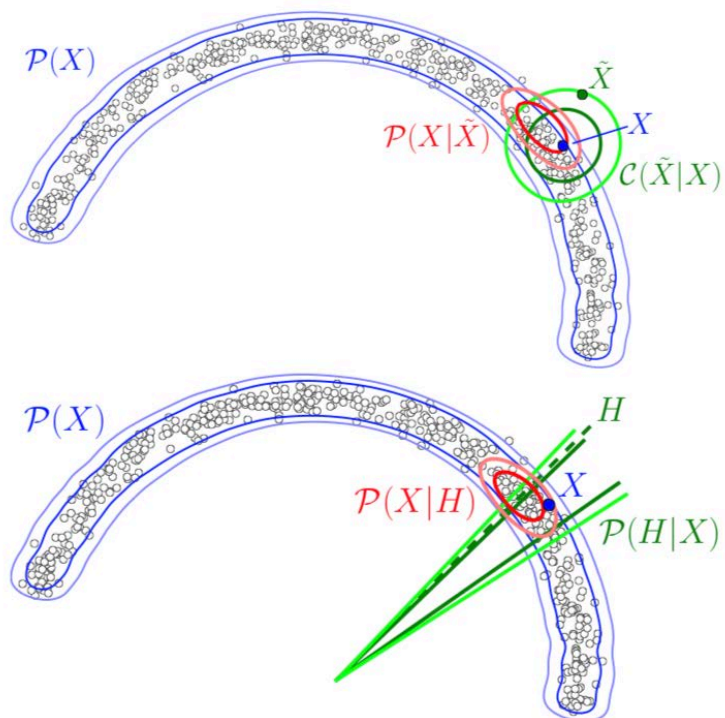
instead of using an *explicit* probability density,  
learn a model that defines an *implicit density*



specify a stochastic procedure for generating the data  
that does not require an explicit likelihood evaluation



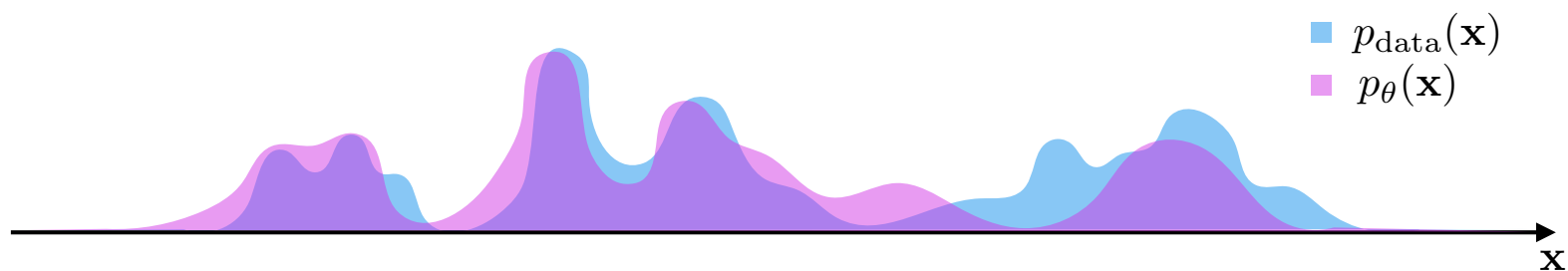
# Generative Stochastic Networks (GSNs)



Deep Generative Stochastic Networks Trainable by Backprop, Bengio et al., 2013

train an auto-encoder to learn Monte Carlo sampling transitions  
the generative distribution is *implicitly* defined by this transition





estimate density ratio through *hypothesis testing*

data distribution  $p_{\text{data}}(\mathbf{x})$

generated distribution  $p_{\theta}(\mathbf{x})$

$$\frac{p_{\text{data}}(\mathbf{x})}{p_{\theta}(\mathbf{x})} = \frac{p(\mathbf{x}|y = \text{data})}{p(\mathbf{x}|y = \text{model})}$$

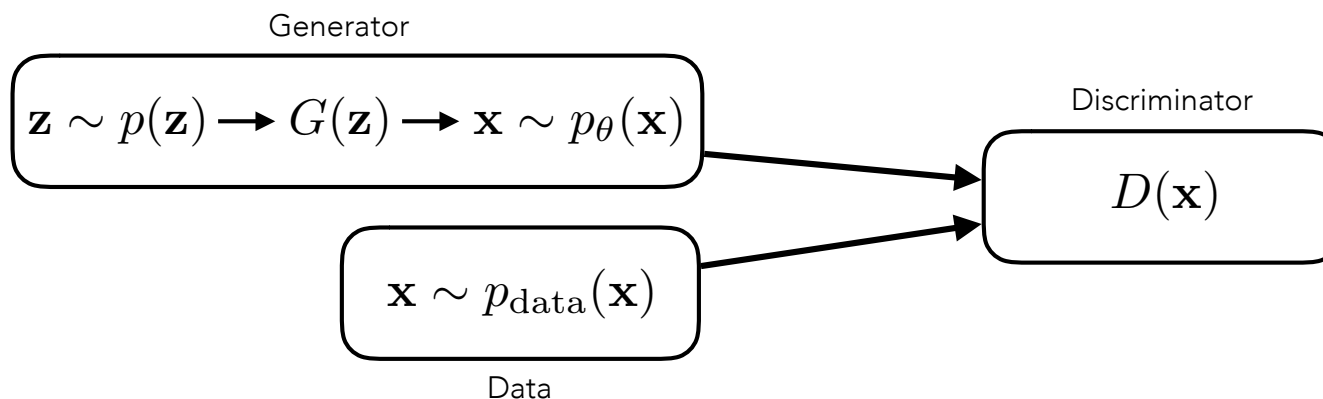
$$\frac{p_{\text{data}}(\mathbf{x})}{p_{\theta}(\mathbf{x})} = \frac{p(y = \text{data}|\mathbf{x})p(\mathbf{x})/p(y = \text{data})}{p(y = \text{model}|\mathbf{x})p(\mathbf{x})/p(y = \text{model})} \quad (\text{Bayes' rule})$$

$$\frac{p_{\text{data}}(\mathbf{x})}{p_{\theta}(\mathbf{x})} = \frac{p(y = \text{data}|\mathbf{x})}{p(y = \text{model}|\mathbf{x})} \quad (\text{assuming equal dist. prob.})$$

density estimation becomes a sample discrimination task



# Generative Adversarial Networks (GANs)



Generator:  $G(\mathbf{z})$

Discriminator:  $D(\mathbf{x}) = \hat{p}(y = \text{data}|\mathbf{x}) = 1 - \hat{p}(y = \text{model}|\mathbf{x})$

Log-Likelihood:  $\mathbb{E}_{p_{\text{data}}(\mathbf{x})} [\log \hat{p}(y = \text{data}|\mathbf{x})] + \mathbb{E}_{p_{\theta}(\mathbf{x})} [\log \hat{p}(y = \text{model}|\mathbf{x})]$

$$= \mathbb{E}_{p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{p_{\theta}(\mathbf{x})} [\log(1 - D(\mathbf{x}))]$$
$$= \mathbb{E}_{p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{p(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

**Minimax:**  $\min_G \max_D \mathbb{E}_{p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{p(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$



# Generative Adversarial Networks (GANs)

**Minimax:** 
$$\min_G \max_D \mathbb{E}_{p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{p(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

GANs minimize the Jensen-Shannon Divergence:

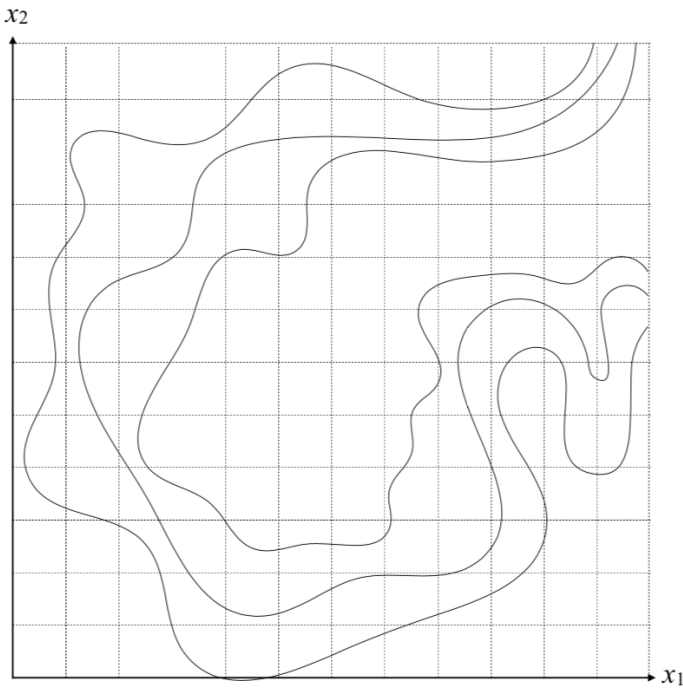
For a fixed  $G(\mathbf{z})$  the optimal discriminator is  $D^*(\mathbf{x}) = \frac{p_{\text{data}}(\mathbf{x})}{p_{\text{data}}(\mathbf{x}) + p_{\theta}(\mathbf{x})}$

Plugging this into the objective

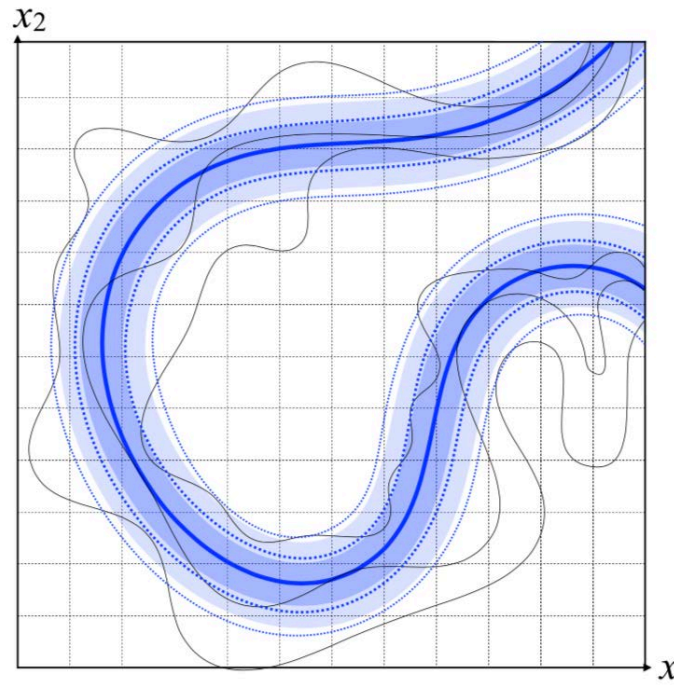
$$\begin{aligned} & \mathbb{E}_{p_{\text{data}}(\mathbf{x})} [\log D^*(\mathbf{x})] + \mathbb{E}_{p_{\theta}(\mathbf{x})} [\log(1 - D^*(\mathbf{x}))] \\ &= \mathbb{E}_{p_{\text{data}}(\mathbf{x})} \left[ \log \left( \frac{p_{\text{data}}(\mathbf{x})}{p_{\text{data}}(\mathbf{x}) + p_{\theta}(\mathbf{x})} \right) \right] + \mathbb{E}_{p_{\theta}(\mathbf{x})} \left[ \log \left( \frac{p_{\theta}(\mathbf{x})}{p_{\text{data}}(\mathbf{x}) + p_{\theta}(\mathbf{x})} \right) \right] \\ &= \log \left( \frac{1}{4} \right) + D_{KL} \left( p_{\text{data}}(\mathbf{x}) \left\| \frac{p_{\text{data}}(\mathbf{x}) + p_{\theta}(\mathbf{x})}{2} \right\| \right) + D_{KL} \left( p_{\theta}(\mathbf{x}) \left\| \frac{p_{\text{data}}(\mathbf{x}) + p_{\theta}(\mathbf{x})}{2} \right\| \right) \\ &= \log \left( \frac{1}{4} \right) + 2 \cdot D_{JS}(p_{\text{data}}(\mathbf{x}) \| p_{\theta}(\mathbf{x})) \end{aligned}$$



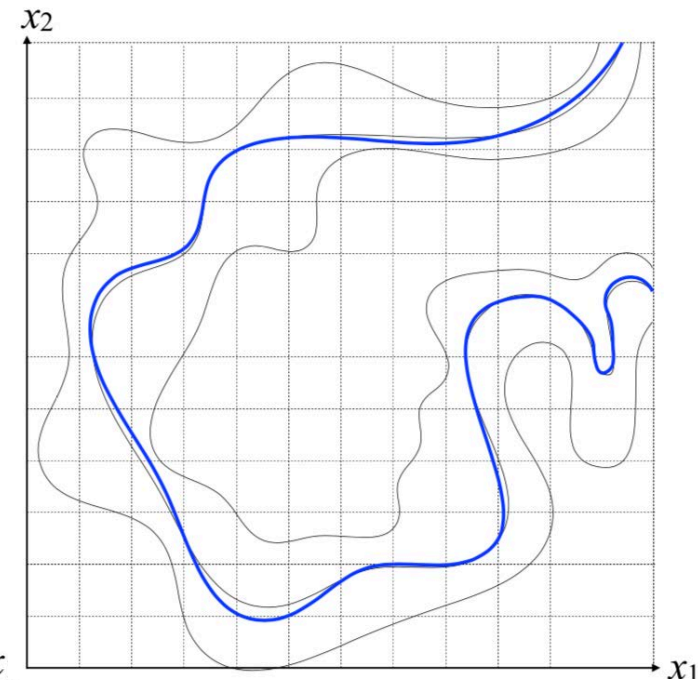
## interpretation



data manifold



explicit model



implicit model

explicit models tend to cover the entire data manifold, but are constrained

implicit models tend to capture part of the data manifold, but can neglect other parts

→ "mode collapse"



# Generative Adversarial Networks (GANs)

GANs can be difficult to optimize

DCGAN	LSGAN	WGAN (clipping)	WGAN-GP (ours)
Baseline ( $G$ : DCGAN, $D$ : DCGAN)			
			
$G$ : No BN and a constant number of filters, $D$ : DCGAN			
			
$G$ : 4-layer 512-dim ReLU MLP, $D$ : DCGAN			
			
No normalization in either $G$ or $D$			
			
Gated multiplicative nonlinearities everywhere in $G$ and $D$			
			
tanh nonlinearities everywhere in $G$ and $D$			
			
101-layer ResNet $G$ and $D$			
			

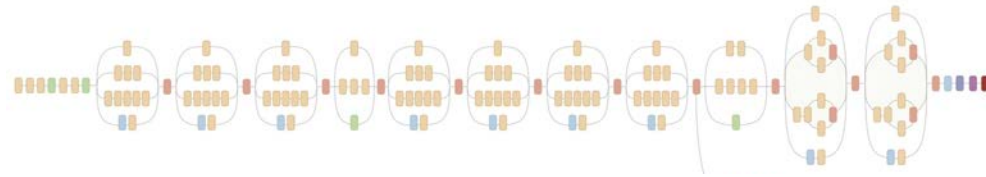
Improved Training of Wasserstein GANs, Gulrajani et al., 2017



# evaluation

without an explicit likelihood, it is difficult to quantify the performance

## inception score



use a pre-trained Inception v3 model to quantify class and distribution entropy

$$\text{IS}(G) = \exp \left( \mathbb{E}_{p(\tilde{\mathbf{x}})} D_{KL}(p(y|\tilde{\mathbf{x}}) || p(y)) \right)$$

$p(y|\tilde{\mathbf{x}})$  is the class distribution for a given image

→ should be highly peaked (low entropy)

$p(y) = \int p(y|\tilde{\mathbf{x}}) d\tilde{\mathbf{x}}$  is the marginal class distribution

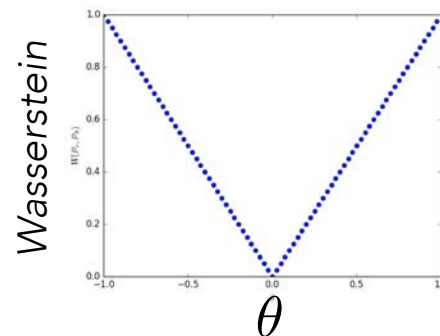
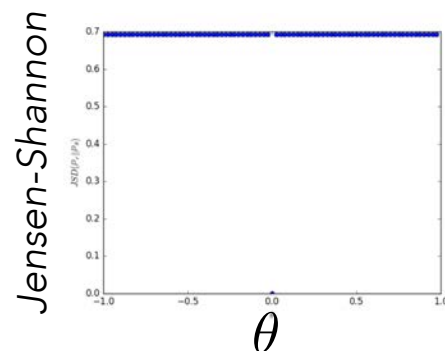
→ want this to be uniform (high entropy)



# Wasserstein GAN (W-GAN)

the Jensen-Shannon divergence can be discontinuous, making it difficult to train

$\theta$  is a gen. model parameter



instead use the Wasserstein distance, continuous and diff. almost everywhere:

$$W(p_{\text{data}}(\mathbf{x}), p_{\theta}(\mathbf{x})) = \inf_{\gamma \in \Pi(p_{\text{data}}(\mathbf{x}), p_{\theta}(\mathbf{x}))} \mathbb{E}_{(\hat{\mathbf{x}}, \tilde{\mathbf{x}}) \sim \gamma} [||\hat{\mathbf{x}} - \tilde{\mathbf{x}}||]$$

*"minimum cost of transporting points between two distributions"*

intractable to evaluate, but can instead constrain the discriminator

$$\min_G \max_{D \in \mathcal{D}} \mathbb{E}_{p_{\text{data}}(\mathbf{x})} [D(\mathbf{x})] - \mathbb{E}_{p_{\theta}(\mathbf{x})} [D(\mathbf{x})]$$

$\mathcal{D}$  is the set of Lipschitz functions (bounded derivative),  
enforced through weight clipping, gradient penalty, spectral normalization

**Wasserstein GANs**, Arjovsky et al., 2017

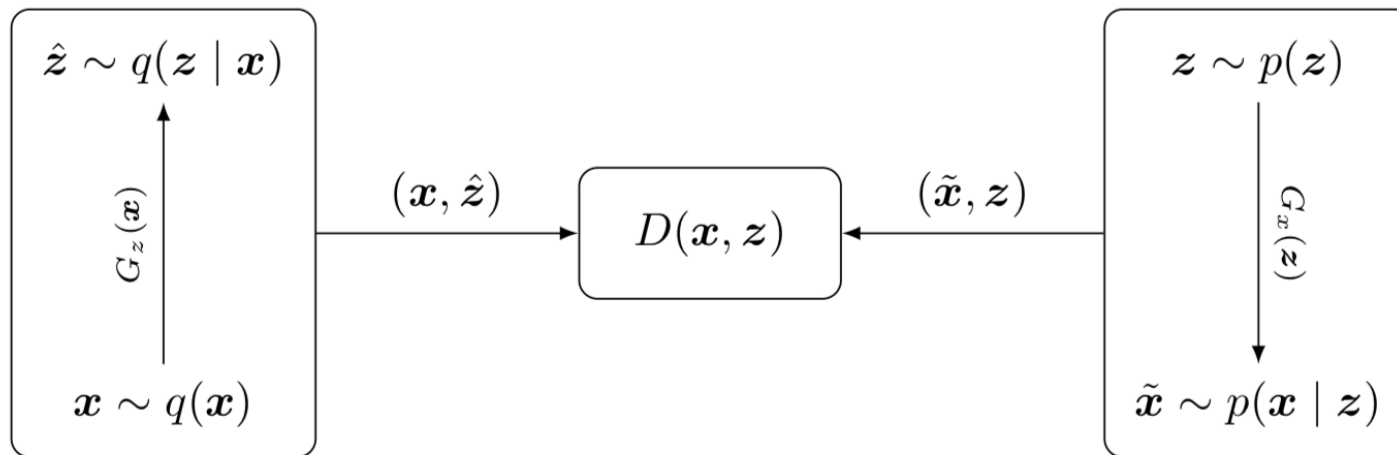
**Improved Training of Wasserstein GANs**, Gulrajani et al., 2017

**Spectral Normalization for GANs**, Miyato et al., 2018

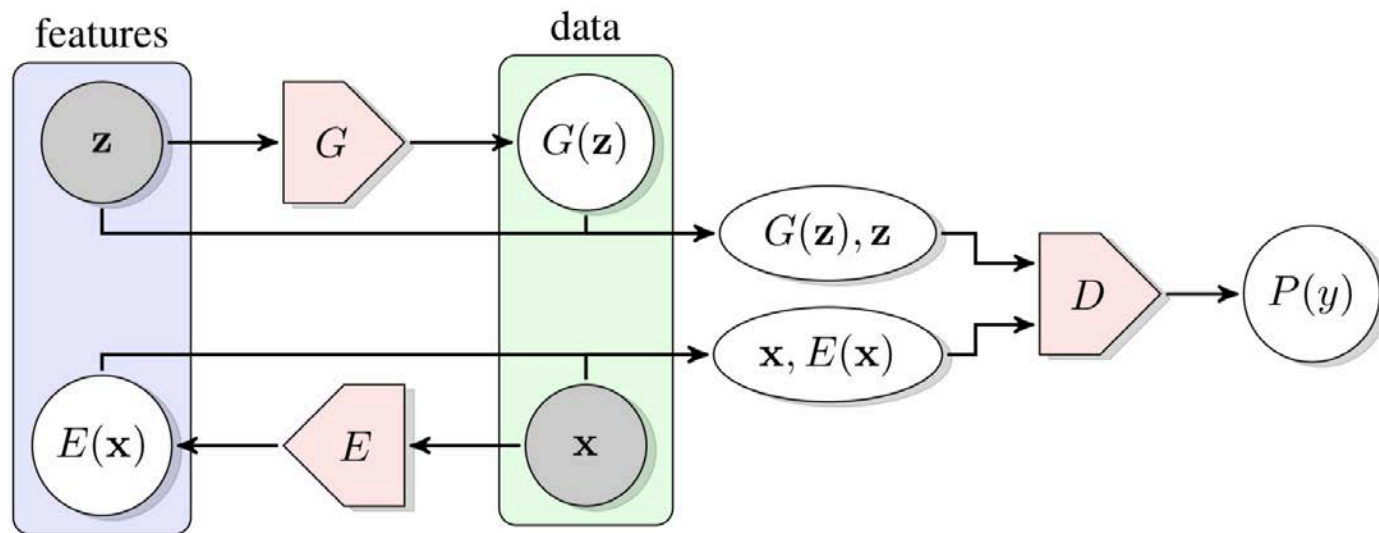


## extensions: inference

can we also learn to **infer a latent representation**?



Adversarially Learned Inference, Dumoulin et al., 2017



Adversarial Feature Learning, Donahue et al., 2017



# applications

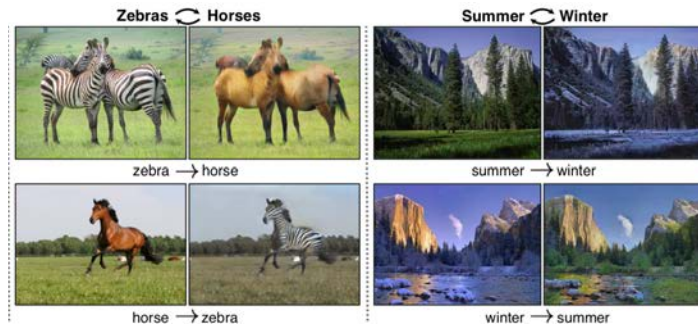
## image to image translation



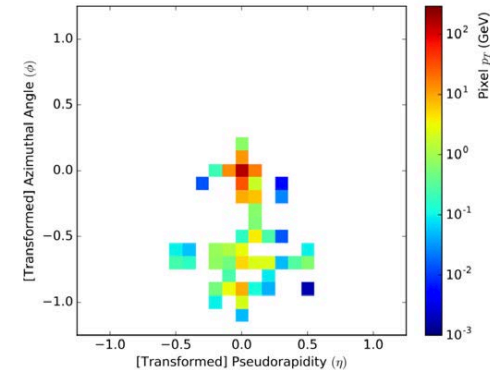
**Image-to-Image Translation with Conditional Adversarial Networks**, *Isola et al.*, 2016



**Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks**, *Zhu et al.*, 2017

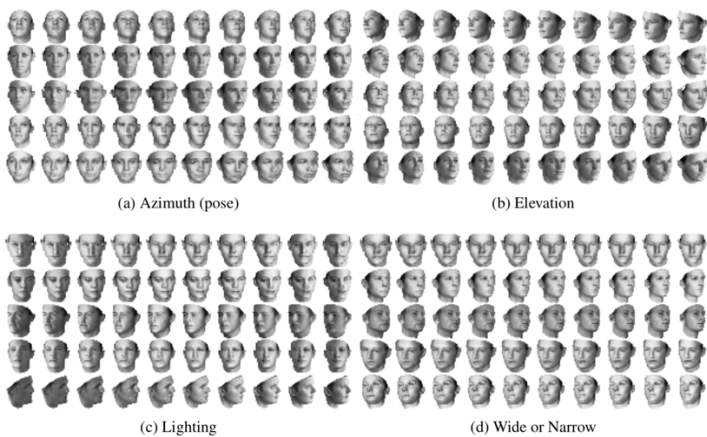


## experimental simulation



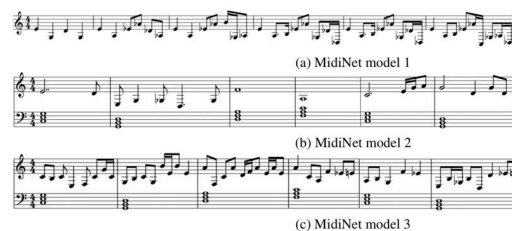
**Learning Particle Physics by Example**, *de Oliveira et al.*, 2017

## interpretable representations



**InfoGAN: Interpretable Representation Learning by Information Maximizing Generative Adversarial Nets**, *Chen et al.*, 2016

## music synthesis



**MIDINET: A CONVOLUTIONAL GENERATIVE ADVERSARIAL NETWORK FOR SYMBOLIC-DOMAIN MUSIC GENERATION**, *Yang et al.*, 2017

## text to image synthesis



**StackGAN: Text to Photo-realistic Image Synthesis with Stacked Generative Adversarial Networks**, *Zhang et al.*, 2016





2014



2015



2016



2017



2018

[arxiv.org/abs/1406.2661](https://arxiv.org/abs/1406.2661)  
[arxiv.org/abs/1511.06434](https://arxiv.org/abs/1511.06434)  
[arxiv.org/abs/1606.07536](https://arxiv.org/abs/1606.07536)  
[arxiv.org/abs/1710.10196](https://arxiv.org/abs/1710.10196)  
[arxiv.org/abs/1812.04948](https://arxiv.org/abs/1812.04948)

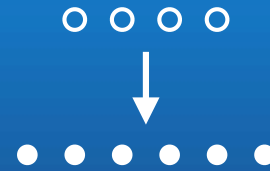


# DISCUSSION

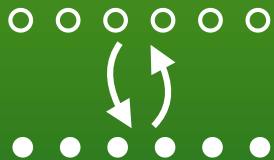




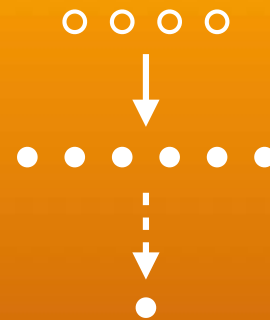
*autoregressive  
models*



*explicit  
latent variable models*



*invertible explicit  
latent variable models*

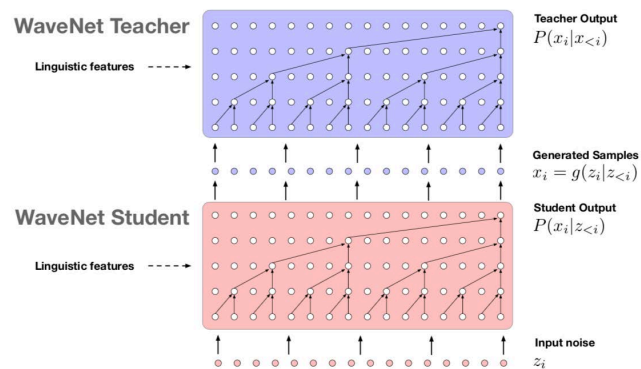


*implicit  
latent variable models*



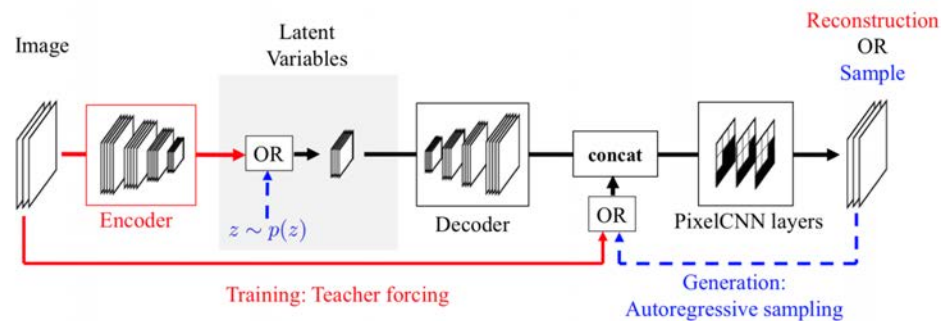
# combining models

## autoregressive + invertible model



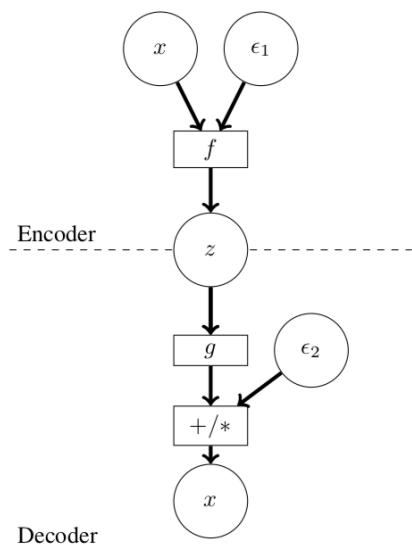
**Parallel WaveNet**, van den Oord et al., 2018

## autoregressive + explicit latent variable model



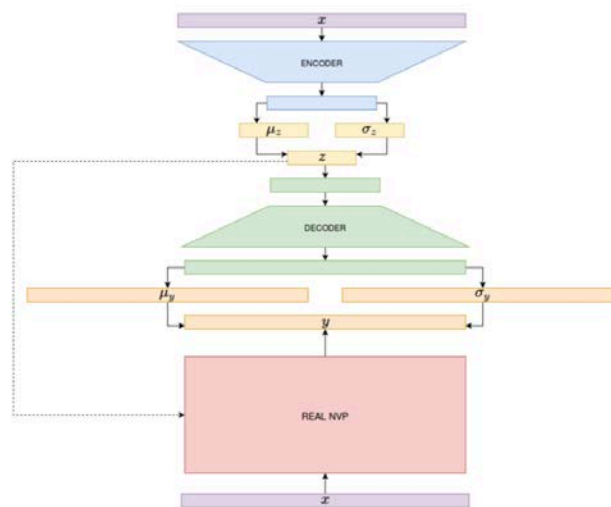
**PixelVAE**, Gulrajani et al., 2017

## explicit + implicit latent variable model



**Adversarial Variational Bayes**, Mescheder et al., 2017

## explicit + invertible latent variable model



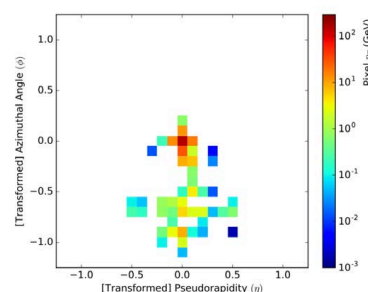
**Deep Variational Inference Without Pixel-Wise Reconstruction**, Agrawal & Dukkupati, 2016



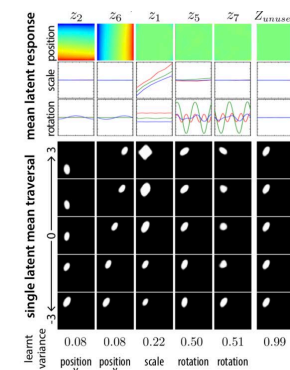
# generative models: *what are they good for?*

generative models model the data distribution

## 1. can generate and simulate data



## 2. can extract structure from data





# ethical concerns



StyleGAN, Karras et al., 2018



# ethical concerns

SYSTEM PROMPT  
(HUMAN-WRITTEN)

*A train carriage containing controlled nuclear materials was stolen in Cincinnati today. Its whereabouts are unknown.*

MODEL COMPLETION  
(MACHINE-WRITTEN,  
FIRST TRY)

The incident occurred on the downtown train line, which runs from Covington and Ashland stations.

In an email to Ohio news outlets, the U.S. Department of Energy said it is working with the Federal Railroad Administration to find the thief.

“The theft of this nuclear material will have significant negative consequences on public and environmental health, our workforce and the economy of our nation,” said Tom Hicks, the U.S. Energy Secretary, in a statement. “Our top priority is to secure the theft and ensure it doesn’t happen again.”

The stolen material was taken from the University of Cincinnati’s Research Triangle Park nuclear research site, according to a news release from Department officials.

The Nuclear Regulatory Commission did not immediately release any information.

According to the release, the U.S. Department of Energy’s Office of Nuclear Material Safety and Security is leading that team’s investigation.

“The safety of people, the environment and the nation’s nuclear stockpile is our highest priority,” Hicks said. “We will get to the bottom of this and make no excuses.”



# ethical concerns



Login

Startups  
Apps  
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Podcasts  
Extra Crunch  
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Events  
Advertise  
Crunchbase  
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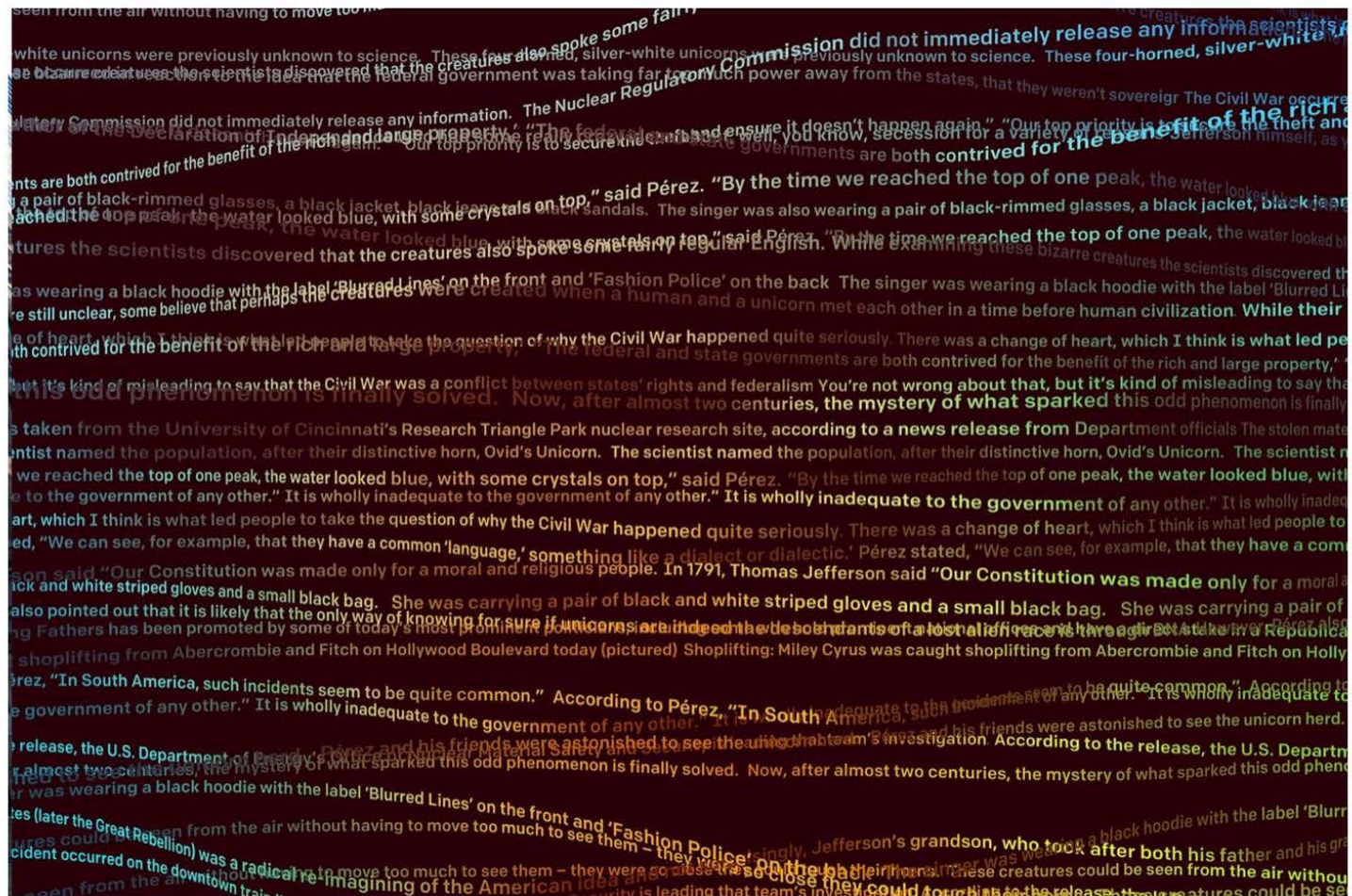
Cybersecurity 101  
Google  
Transportation  
Asia

Search

## OpenAI built a text generator so good, it's considered too dangerous to release

Zack Whittaker @zackwhittaker / 2 weeks ago

Comment



A storm is brewing over a new language model, built by non-profit artificial intelligence research company

**OpenAI**, which it says is so good at generating convincing, well-written text that it's worried about potential abuse.



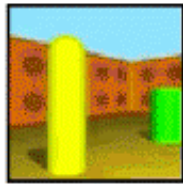
applying generative models to new forms of data



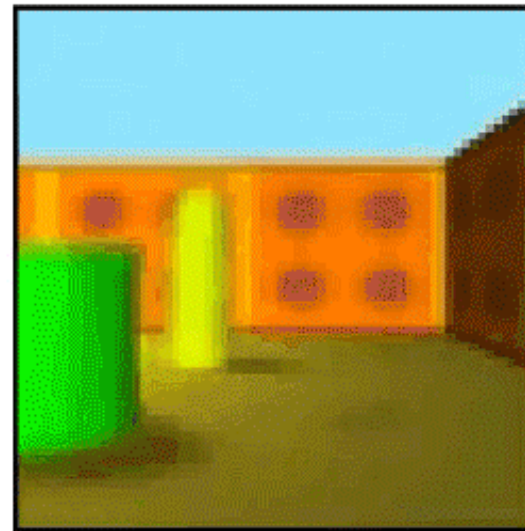


model-based RL: using a (generative) model to plan actions

observation



neural rendering



**GQN**, Eslami *et al.*, 2018



