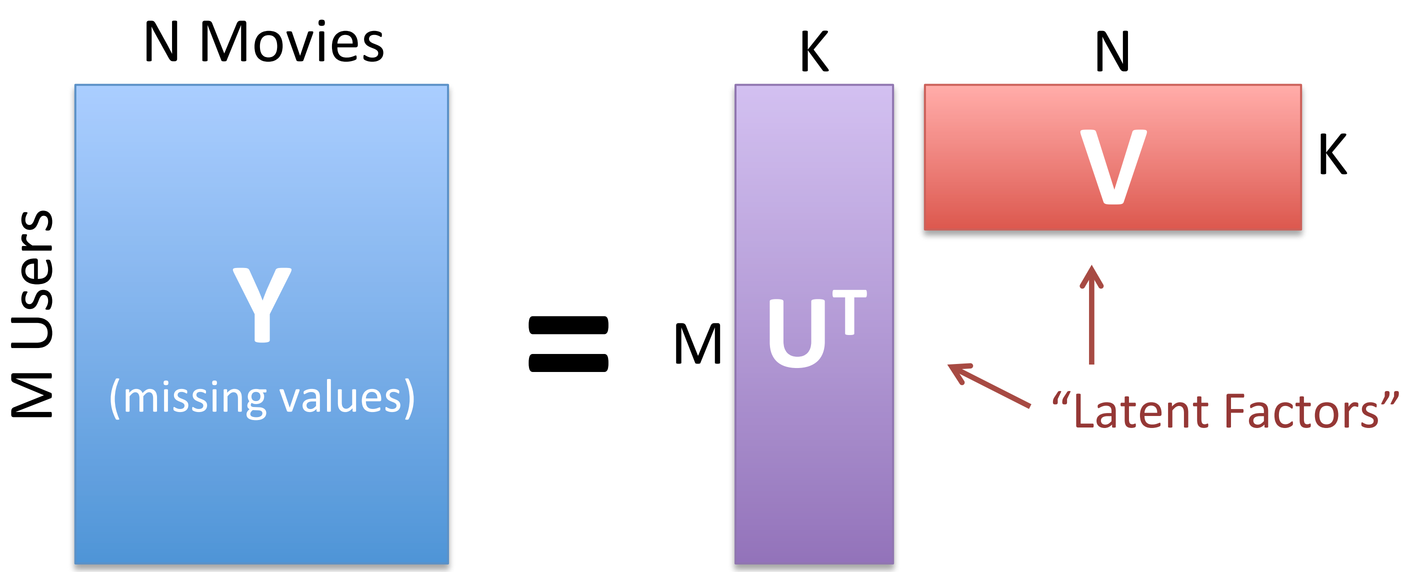
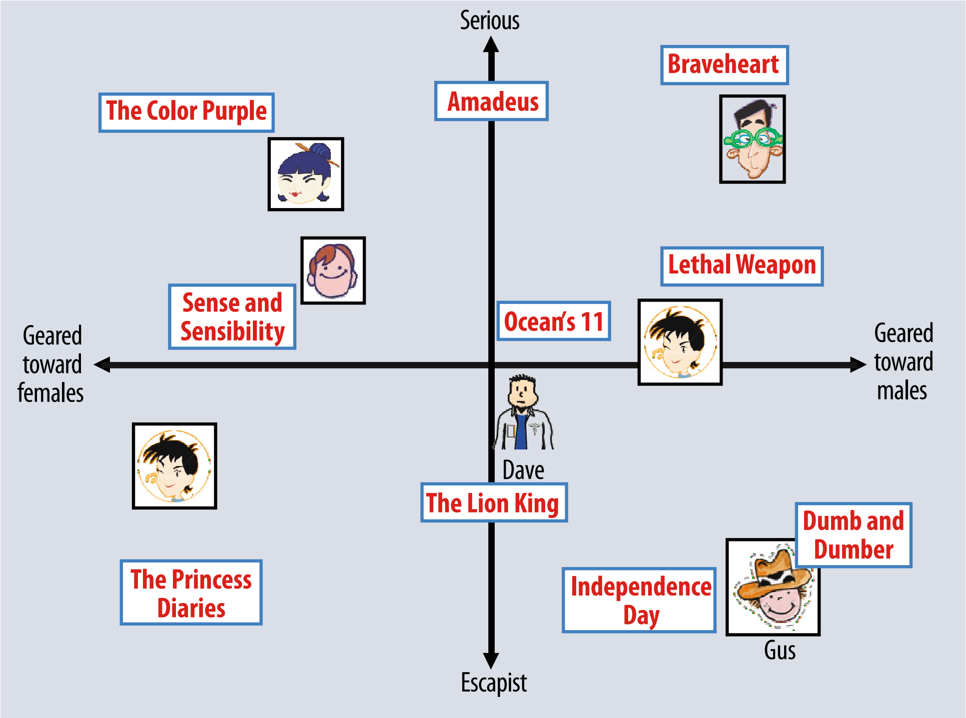
Step-By-Step Instructions for Miniproject 2

Matrix Factorization with Missing Values



- Goal #1: Learn a Latent Factor Model U & V
- Goal #2: Visualize & Interpret U & V (mostly V)

Final Product: Create Something Like This



(Your visualization will probably not be as clean as this one, that is OK)

You need to create your own visualization (will have different projection of movies/users onto 2-dimensional plane than example above)

You need to interpret your dimensions and/or clusters of movies in your projection

http://www2.research.att.com/~volinsky/papers/ieeecomputer.pdf

Outline

- Step 1: Learn U & V
  - If you are not confident in implementation, use off-the-shelf software first
  - Then implement your own solver if you feel like it
- Step 2: Project U & V down to 2 dimensions
  - Basically SVD in Matlab or Python
- Step 3: Plot projected U & V
  - Give your own interpretation of the two projected dimensions

Step 1: Learning U & V

Choice of regularization doesn't matter too much

You don't have to solve this exact objective.
(many off-the-shelf solve something related.)

$$\underset{U,V}{\operatorname{argmin}} \frac{\lambda}{2}\left(\|U\|^2 + \|V\|^2\right) + \sum_{(i,j)\in S}\left(Y_{i,j} - u_i^T v_j\right)^2$$

S = set of indices (i,j)
of observed ratings

- Three options:
  - Stochastic Gradient Descent
    - Each step of SGD considers single index (i,j) of S
  - Alternating Minimization
    - Each step completely solves U or V while holding the other fixed.
  - Use off-the-shelf software
    - Will only get 20/40 of this portion of the question

Off-the-Shelf Software

- Search for "Collaborative Filtering Matlab" or "Collaborative Filtering Python" or "Collaborative Filtering code"

- http://bickson.blogspot.com/2012/12/collaborative-filtering-with-graphchi.html
- http://spark.apache.org/docs/1.0.0/mllib-collaborative-filtering.html
- http://select.cs.cmu.edu/code/graphlab/pmf.html
- http://www.quuxlabs.com/blog/2010/09/matrix-factorization-a-simple-tutorial-and-implementation-in-python/

Step 1b: Learning U & V
(More Advanced)

Choice of regularization doesn't matter too much

$$\underset{U,V,a,b}{\operatorname{argmin}} \frac{\lambda}{2}\left(\|U\|^2 + \|V\|^2\right) + \sum_{(i,j)\in S}\left(Y_{i,j} - \left(u_i^T v_j + a_i + b_j\right)\right)^2$$

S = set of indices (i,j)
of observed ratings

Vector of bias/offset terms
One for each user & movie

- Model the global tendency of a movie's average rating
- Model the global tendency of how a user rates on average
- This keeps U & V more focused on variability between users and movies.
- Should be an option that you can turn on in many off-the-shelf implementations

Step 1c: Learning U & V
(Even More Advanced)

Choice of regularization doesn't matter too much

μ is average of all observations in Y

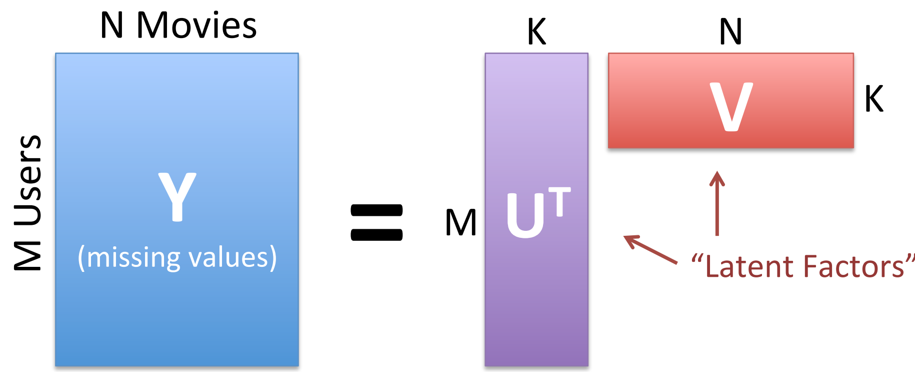
$$\underset{U,V,a,b}{\operatorname{argmin}} \frac{\lambda}{2}\left(\|U\|^2 + \|V\|^2 + \|a\|^2 + \|b\|^2\right) + \sum_{(i,j)\in S}\left(\left(Y_{i,j} - \mu\right) - \left(u_i^T v_j + a_i + b_j\right)\right)^2$$

S = set of indices (i,j)
of observed ratings

Vector of bias/offset terms
One for each user & movie

- Model global bias μ as average over all observed Y
- Treat a as user-specific deviation from global bias
- Treat b as movie-specific deviation from global bias
- Should be an option that you can turn on in many off-the-shelf implementations

Step 1: Interpretation



- Common K-dimensional representation over users & movies
  - Rating defined by dot product (aka un-normalized cosine similarity):

$$Y_{i,j} \approx u_i^T v_j \quad \text{or} \quad Y_{i,j} \approx u_i^T v_j + a_i + b_j$$

- Does our representation make sense? (i.e., is it interpretable?)
  - Need to visualize!
  - But can only (easily) visualize 2-dim points, not K-dim points!

Step 2: Projecting U & V to 2 Dimensions

- Step 2a:
  - (Optional) mean center V: each row of V has zero mean
  - Compute SVD of V:

$$V = \underbrace{A}_{\text{Orthogonal}}\underbrace{\Sigma}_{\text{Diagonal}}\underbrace{B^T}_{\text{Orthogonal}}$$

  - The first two columns of A correspond to best 2-dimensional projection of movies V

Step 2: Projecting U & V to 2 Dimensions

- Step 2b:
  - Project every movie & user using $A_{1:2}$

$$\tilde{V} = A_{1:2}^T V \in \mathrm{Re}^{2\times N}$$

$$\tilde{U} = A_{1:2}^T U \in \mathrm{Re}^{2\times M}$$

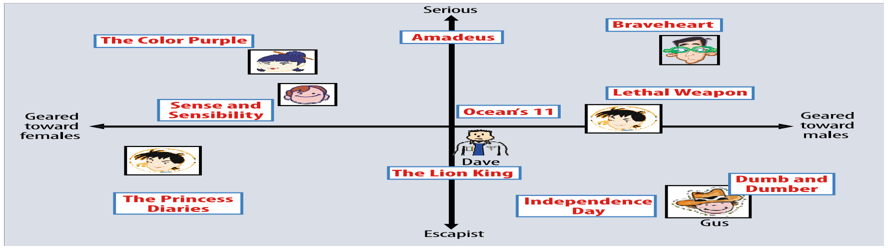
If you mean centered V, you need to shift U by same amount first

  - Now each user & movie is represented using a two dimensional point. Visualize and interpret!

Step 2: Projecting U & V to 2 Dimensions

- Step 2c (optional):
  - Do Steps 2a & 2b: $\tilde{V} = A_{1:2}^T V \in \mathrm{Re}^{2\times N}$

    $\tilde{U} = A_{1:2}^T U \in \mathrm{Re}^{2\times M}$
  - Then rescale dimensions:
    - E.g., each row of Ũ has unit variance.
    - Otherwise, visualization might look stretched:



Step 2: Interpretation

- The top D dimensions of matrix A defines a D-dim projection that best preserves the learned movie features V:

$$\tilde{v}_j = A_{1:D}^T v_j$$

Minimizes loss of feature representation:

$$\sum_j \left\| v_j - A_{1:D} \tilde{v}_j \right\|^2$$

Projected representation

Preservation Loss of projection

- We want 2-dimensional projection for visualization purposes
  - So we take top 2 dimensions of SVD
- Now we can visualize movies in 2D plot
  - And see if close-by movies have similar sementics
  - E.g., horror, action, etc.

Step 2: Alternatives & Core Requirements

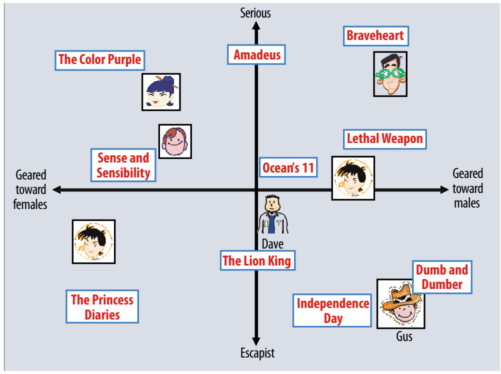
- You don't have to do it the above way
  - Although the above method should always give you something reasonable to visualize

- Core requirement:
  - Projection should preserves as much of the original features as possible
  - A dot product in the 2-D representation should approximate the dot product in the K-D representation

Step 3: Plot U & V

- Plotting V is more important:
  - Pick a few movies and plot their projected 2D representation
  - Verify that distances/angles/axes in your plot can be interpreted

Example:



(Your visualization will probably not be as clean as this one, that is OK)

- Can also plot the genres provided:
  - E.g., where is the average horror movie?
  - E.g., compute the average v for all movies that belong to horror genre

My Own Example

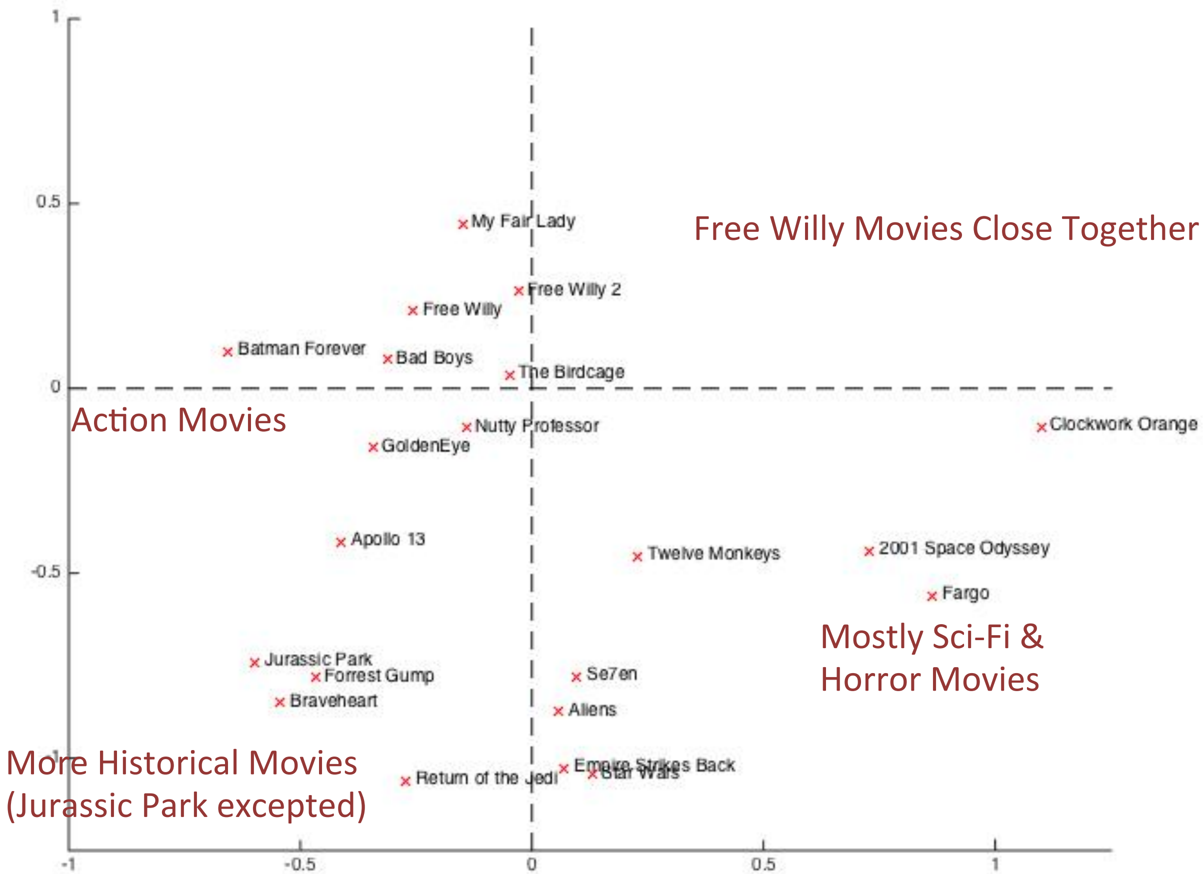
Trained using
Step 1c (lambda=10)
Stochastic GD

SVD of Movie Matrix
Project top 2 bases

Picked a few popular movies, and plotted them.

Then found a few extreme points (e.g., Clockwork Orange).

Removed most children's movies (didn't seem to project well using 1st two SVD bases – maybe most ratings are by adults).



Star Wars Movies
Close together